

DYNAMIC BEHAVIOUR OF  
SET BACK STRUCTURES

Sudhanshu Kumar Varma

A MAJOR TECHNICAL REPORT

in

The Faculty

of

Engineering

Presented in Partial Fulfillment of the Requirements  
for the Degree of Master of Engineering at  
Sir George Williams University  
Montreal, Canada

July, 1974

## ABSTRACT

### DYNAMIC BEHAVIOUR OF SETBACK STRUCTURES

BY SUDHANSHU K. VARMA

The purpose of this study is to investigate the effect of symmetric setback on the dynamic behaviour of a tall building with setback based on (i) Seismic Building Code, (ii) Elastic Analysis and (iii) Elasto Plastic Analysis. Two problems arise due to the presence of setback in a building. First there is serious concentration of lateral shear force in the vicinity of the setback. Secondly if the setback is unsymmetrical it destroys the Dynamic Symmetry of the building and translational as well as torsional vibrations may be induced.

The present study is restricted to buildings of a rectangular cross section having setback at only one level along their height. Setback is defined by two parameters, one referring to the level and the other to the size or degree of the setback. Only horizontal motions are considered. Response is studied for lumped mass models subjected to high intensity E.Q. excitation for both linearly elastic and bilinear member behaviour. Viscous damping is assumed to be 2%. Code recommendations are evaluated and comparisons made.

The following conclusions may be drawn from the study reported here.

- The fundamental translational period of a building is lowered by decrease in the level of setback and/or degree of setback.

- Modal displacements undergo an abrupt change at setback level.

Modal displacements of structures with large towers are found to be comparable to uniform structures.

- The code procedure underestimates the shear response of yielding structures whereas elastic analysis overestimates the response. - The variation of ratio of inelastic base shear to code base shear is found to be in the range of 1.5 to 5.0 (structures with relatively stiff tall towers yield the highest ratio). The ratio of elastic to inelastic base shear is close to 2.
  - Interstorey sway may show an abrupt increase at setback level. In-elastic analysis leads to high degree of change in small towers whereas with elastic analysis taller towers show high degree of change.
  - Elastic sway is slightly greater than inelastic sway and generally no abrupt changes are noticed at setback level.
  - Abrupt changes in girder and column ductility factors are found at setback level, max<sup>m</sup> ductility occurring in relatively stiff tall towers. Top and bottom floors possess roughly similar girder ductility factors.
- 
- Column ductility factors are found to be greater in the tower than the base and also in less stiff setback structures.

No significant difference in girder ductility factors is noticed while comparing elastic and inelastic member behaviour. However column ductility factors increase due to inelastic behaviour.

#### ACKNOWLEDGEMENTS

The author is sincerely grateful to his teacher and guide Dr. O. A. Pekau for suggesting the problem and providing continuous guidance and encouragement in the course of this study.

He also wishes to thank Miss Valentina Szyroka of Electrical Engineering Dept. for typing the report.

## TABLE OF CONTENTS

Abstract . . . . .	i
Acknowledgements . . . . .	iii
List of Figures . . . . .	vi
List of Tables . . . . .	v
SECTION 1 INTRODUCTION . . . . .	1
1.1 Problem . . . . .	1
1.2 Related Studies . . . . .	2
1.3 Scope of Present Study . . . . .	4
SECTION 2 SELECTION OF MODEL FRAMES . . . . .	6
SECTION 3 ANALYTICAL INVESTIGATION . . . . .	14
3.1 Investigation Criteria . . . . .	14
3.2 Parameters . . . . .	15
3.3 Code Shear Response . . . . .	18
3.4 Calculation of Shear Response . . . . .	23
SECTION 4 RESULTS & DISCUSSION . . . . .	32
4.1 Fundamental Period . . . . .	32
4.2 Mode Shapes . . . . .	33
4.3 Total Storey Shear . . . . .	34
4.4 Absolute Acceleration . . . . .	35
4.5 Inter Storey Sway . . . . .	36
4.6 Total Sway . . . . .	37
4.7 Column Ductility . . . . .	37
4.8 Girder Ductility . . . . .	38
4.9 Graphical Representation of Results . . . . .	38
CONCLUSION . . . . .	38
References . . . . .	84

## LIST OF TABLES

Table 2.1	MEMBER PROPERTIES, UNIFORM MODEL . . . . .	11
Table 2.2	ADJUSTMENT OF REFERENCE RIGIDITY . . . . .	13
Table 3.1	SUMMARY OF CODE SHEAR RESPONSE . . . . .	28
Table 3.2	COMPARATIVE SUMMARY OF SHEAR RESPONSE . . . . .	29

# LIST OF FIGURES

Fig. 2.1	FRAME GEOMETRY . . . . .	8
Fig. 2.2	MODEL DESIGNATION & GEOMETRY . . . . .	10
Fig. 3.1	INTRODUCTION OF DUMMY MEMBERS . . . . .	16
Fig. 3.2	SET-BACK STRUCTURE ELEVATION . . . . .	22
Fig. 4.1	GRAPH PERIOD vs $l_s$ . . . . .	40
Fig. 4.2	GRAPH PERIOD vs $C_s$ . . . . .	41
Fig. 4.3.A	MODE SHAPES, ( $C_s = 0.667$ ) . . . . .	42
Fig. 4.3.B	MODE SHAPES, ( $C_s = 0.333$ ) . . . . .	43
Fig. 4.3.C	MODE SHAPES, ( $C_s = 0.167$ ) . . . . .	44
Fig. 4.4.A	TOTAL STOREY SHEAR, ( $C_s = 0.667$ ) . . . . .	45
Fig. 4.4.B	TOTAL STOREY SHEAR, ( $C_s = 0.333$ ) . . . . .	48
Fig. 4.4.C	TOTAL STOREY SHEAR, ( $C_s = 0.167$ ) . . . . .	51
Fig. 4.5.A	ABSOLUTE ACCELERATION, ( $C_s = 0.667$ ) . . . . .	54
Fig. 4.5.B	ABSOLUTE ACCELERATION, ( $C_s = 0.333$ ) . . . . .	56
Fig. 4.5.C	ABSOLUTE ACCELERATION, ( $C_s = 0.167$ ) . . . . .	58
Fig. 4.6.A	INTER STOREY SWAY, ( $C_s = 0.667$ ) . . . . .	60
Fig. 4.6.B	INTER STOREY SWAY, ( $C_s = 0.333$ ) . . . . .	62
Fig. 4.6.C	INTER STOREY SWAY, ( $C_s = 0.167$ ) . . . . .	64
Fig. 4.7.A	TOTAL SWAY ( $C_s = 0.667$ ) . . . . .	66
Fig. 4.7.B	TOTAL SWAY ( $C_s = 0.333$ ) . . . . .	68
Fig. 4.7.C	TOTAL SWAY ( $C_s = 0.167$ ) . . . . .	70
Fig. 4.8.A	COLUMN DUCTILITY FACTOR ( $C_s = 0.667$ ) . . . . .	72
Fig. 4.8.B	COLUMN DUCTILITY FACTOR ( $C_s = 0.333$ ) . . . . .	74
Fig. 4.8.C	COLUMN DUCTILITY FACTOR ( $C_s = 0.167$ ) . . . . .	76
Fig. 4.9.A	GIRDER DUCTILITY FACTOR ( $C_s = 0.667$ ) . . . . .	78
Fig. 4.9.B	GIRDER DUCTILITY FACTOR ( $C_s = 0.333$ ) . . . . .	80

Fig. 4.9.C GIRDER DUCTILITY FACTOR ( $C_s = 0.167$ ) . . . . . 82



## NOMENCLATURE

Symbols are defined when they first appear in the text; those which appear frequently are listed below for reference.

C	Base shear coefficient
c	subscript to denote column
$C'_c$	strength ratio for exterior top column to top girder
$C_s$	Degree of setback
E	Young's modulus of Elasticity
F	Force
G	Subscript to denote girder
H	Height
I	Moment of Inertia
K	Numerical coefficient depending upon structure type, its ductility and reserve energy
$l_s$	Level of setback
m	mass
N	Total number of stories
n	storey designation
T	Fundamental Period
V	Base shear
W	Total structure weight
y	subscript to denote yield strength

$\tau_{CG}$  Linear Expression for column and girder properties

$\tau_G$  Variation of girder properties

$\tau_C$  Variation of column properties

$\omega$  Circular frequency

## SECTION 1

### INTRODUCTION

#### 1.1 The Problem

Behaviour of structures with setbacks subjected to earthquake has not received significant study to date, as compared to uniform and regular structures, but the fact remains that setbacks do occur in modern multi-storey buildings in large or small shape and size. Several difficulties arise in the evaluation of effect of setback on the dynamic stresses in a setback structure. First of all it is well known that serious concentration of lateral shear force occurs at the setback level or in the vicinity of setback due to notch effect. Secondly in unsymmetrical setbacks the ground motion induces torsional oscillations in the structure, due to the fact that dynamic symmetry of a structure in one or both principal directions is destroyed.

The method of dealing with the problem in some current seismic building codes is to ignore the effect of setback if the plan area of tower falls in a certain percentage range of the base area (75% or more) in case of codes (1), (2), (3), (4) and (5). For other conditions of setbacks the tower is considered as a separate building for its own height or as a part of the overall structure whichever gives the critical result. SEAOC (1) however requires suitable adjustments depending upon the shape and size of setback, to somewhat approximate the actual dynamic behaviour. The separate tower concept does not take into account the fact that the ground motion is modified to a great extent by the larger base before affecting the tower. Dynamic symmetry in a uniform building may also be

lacking due to unsymmetrical distribution of stiffness, which is usually ignored by codes (1), (2), (3), (4), (5).

### 1.2. Prior Related Studies

Berg<sup>(7)</sup> explored the effect of setbacks using a rectangular stepped cantilever shear beam as a model, considering the general case of coupled lateral-torsional vibration of the beam with unsymmetric setbacks.

Multistorey structures with appendages or very light towers can be considered as extreme cases of setback problem. Penzien<sup>(6)</sup> describes an approximate method of analysis applicable to buildings having offsets or appendages as well as to structures that respond in a combination of torsion and translation, making use of two degree of freedom response spectra of the motion for which the system is being analysed.

The most severe conditions for an appendage arise when its fundamental period of vibration lies close to the rest of the structure. Magnification factors of as much as 8.0 in the appendage's base shear, relative to the result of conventional statical analysis have been reported by Bustamante and Rapoport<sup>(12)</sup>. These very high magnification factors were computed assuming that design response were equal to the root of the sum of modal values. However as shown by Penzien<sup>(6)</sup>, this criterion greatly overestimates the appendage responses when the mass of the appendage is small compared with that of the building (say, less than 1/100 of the building), and there is near coincidence of the fundamental periods of both subsystems.

Jhaveri<sup>(8)</sup> made an extensive study of the problems of setback predicting an elastic member behaviour. The structure is assumed to be linearly elastic and viscous damping is assumed to have negligible effect

on the mode shapes and frequencies of the undamped structure. Horizontal motions are considered and are assumed to be independent of the vertical motions. Study has been reported for the effect of a symmetric setback on the uncoupled translational vibration of a building and also coupled lateral-torsional vibration of a building with unsymmetric setback.

Pekau<sup>(9)</sup> reported a deterministic analysis of structural response of symmetric setback structures in elasto plastic range for digitized histories of ground motion. A bilinear model is assumed for member behaviour and structural response to base excitation is evaluated numerically following an incremental procedure. Damping is assumed of viscous type. Pekau and Green in their paper<sup>(10)</sup> have also examined the effectiveness of using uniform structures, elastic behaviour and simplified code provisions to treat yielding systems and values of the setback parameters where these approximations provide useful results are reported. The results indicate certain ranges of the setback parameters where the effect of yielding and irregular shape do not influence response significantly.

The adequacy of code provisions can be argued based on refs. (6), (7), (8), (9), (10). It is seen from (8) and (9) that code (1) underestimates the shear response (most important for structural design) whereas the analysis based on elastic behaviour (8) tends to overestimate. However it is interesting to note that buildings have survived severe ground motions, possibly designed with an underestimated lateral force. Probable answer is that structures undergo plastic deformation during ground shaking and therefore withstand large shear forces and also due to the force resisting capacity of non-structural members in a modern high rise building.

It may not be always accurate as a rule to depend upon the plastic yielding of the structures, unless the true capacity is explored, rather than treating it as an inherent factor of safety. Furthermore the over-estimated values of elastic analysis can only lead to improper use of material capacity and increased cost and an oversized structure may be more rigid or less ductile.

### 1.3 Scope of Present Study

The purpose of this study is to compare the effect of symmetric setbacks on the dynamic behaviour of a multistorey structure, based on (i) seismic building codes (ii) elastic analysis (iii) elasto plastic analysis. The comparison of response namely shear response may lead to a factor which can be applied to underestimated code shear in order that response may be closely approximated. Comparison of other modal quantities (Mode shapes, Absolute acceleration,  $\text{Max}^m$  sway, Interstorey sway, Girder and Column ductility) may eventually lead to a better comparative understanding of the complex problem.

To explain the fact that a factor is sought than a suggestion that code shears be increased to match exact analysis, it can be said that the behaviour of structure will depend upon the design forces. For argument's sake if high design loads are used, the structure required will be much stiffer and will have a shorter period of vibration than a structure designed in accordance with the code provisions. Because the shorter period of vibration results in higher spectral acceleration, the stiffer structure may be subjected to more lateral force than would the structure designed as code recommends. Consequently, designing for too large a force will not necessarily make the structure safer if in the process structures

become stiffer or less ductile.

The present study is limited to buildings having setback at only one level along their height and having a rectangular cross section both above and below of setback level (called Tower & Base respectively).

It is not intended to develop any new method or mathematical formulation for the analysis but available literatures (8) and (9) have been used to evaluate the response.

Dynamic analyses are made for 13 model setback frame structures. Series of models are generated by varying simple parameters describing tower and base geometry. A bilinear model is assumed for member behaviour, damping is assumed to be of viscous type and structural response to digitized El Centro (N-S Component) excitation is evaluated numerically following an incremental procedure (9). The effect and relative importance of modal response are demonstrated by comparing the inelastic response to corresponding elastic behaviour. Code is compared to above two in shear response only.

Results are demonstrated in the form of graphs (envelopes of maximas) for each sub group of models, and tables. The salient and peculiar features are outlined.

Section 2 is devoted to model selection and their properties. Section 3 describes the steps taken in the analytical investigation of the present study and different parameters affecting the dynamic response are outlined. Base and storey shears are also calculated based on code (1) provisions. Section 4 is devoted to results and discussion on the model behaviour. Also, conclusion is presented in Section 4.

## SECTION 2

## SELECTION OF MODEL FRAMES

For the purpose of this study a number of typical structural frames are developed with varying degree of setback and level of setback.

The series of setback structures studied are shown in Figures 2.1 and 2.2. The Base consists of three equal bays for a fraction of the overall height given by "level of setback  $l_s$ ". The Tower is a single central bay where the sums of girder and column properties at a particular storey equal corresponding storey sums of a uniform three bay structure multiplied by the "degree of setback,  $C_s$ ". REF (9).

Actual values of member properties are obtained from (9), for a typical 10 storey single bay (Design data: Bay width = 20', Equal storey height = 12', D.L = 55<sup>k</sup>, L.L = 37.5<sup>k</sup>, a representative load of residential type building such as hotels), designed for a seismic exposure of zone 3, Ref.(5) and using A.36 steel. This single bay frame has been referred to as standard frame in the text of this study.

The properties of interior columns of setback frames in question are formed by doubling the flexural stiffness and strength of the standard frame. The member flexural rigidity  $EI$  is related to yield moment  $M_y$  by a constant relationship determined for 14 WF section Ref.(9), and the variation of these properties for girders and columns is given by linear expression of storey level  $\frac{z}{z_g}$ ,  $\frac{z}{z_c}$  respectively. Relative column to girder strength is expressed by parameters  $C_c$  giving the strength ratio for the exterior top column to the top girder in uniform structure (Degree of setback  $C_s = 1.0$ ). Therefore from reference (9) the girder and column properties can be summarized as follows:

Girder flexural rigidity  $EI_{Gn} = \gamma_{Gn} EI_o \dots (1)$

Column Yield strength  $M_{yCn} = C_c \gamma_{Cn} M_{yo} \dots (2)$

Column flexural rigidity  $EI_{Cn} = C_c \gamma_{Cn} EI_o \dots (3)$

Girder Yield strength  $M_{yGn} = \gamma_{Gn} M_{yo} \dots (4)$

$$\text{where } \gamma_{Gn} = 1 + \frac{(n-1)(\tau_G - 1)}{N-1}$$

$$\gamma_{Cn} = 1 + \frac{(n-1)(\tau_C - 1)}{N-1}$$

And also from ref. (9)

Reference yield strength  $M_{yo}$  of uniform 3 bay structure in question  $M_{yo} = 1.148 M_{yo} \dots (5)$

where  $M_{yo}$  = Reference strength of standard frame.

Reference rigidity  $EI_o$  of uniform 3 bay structure in question  $EI_o = 0.951 EI_o \dots (6)$

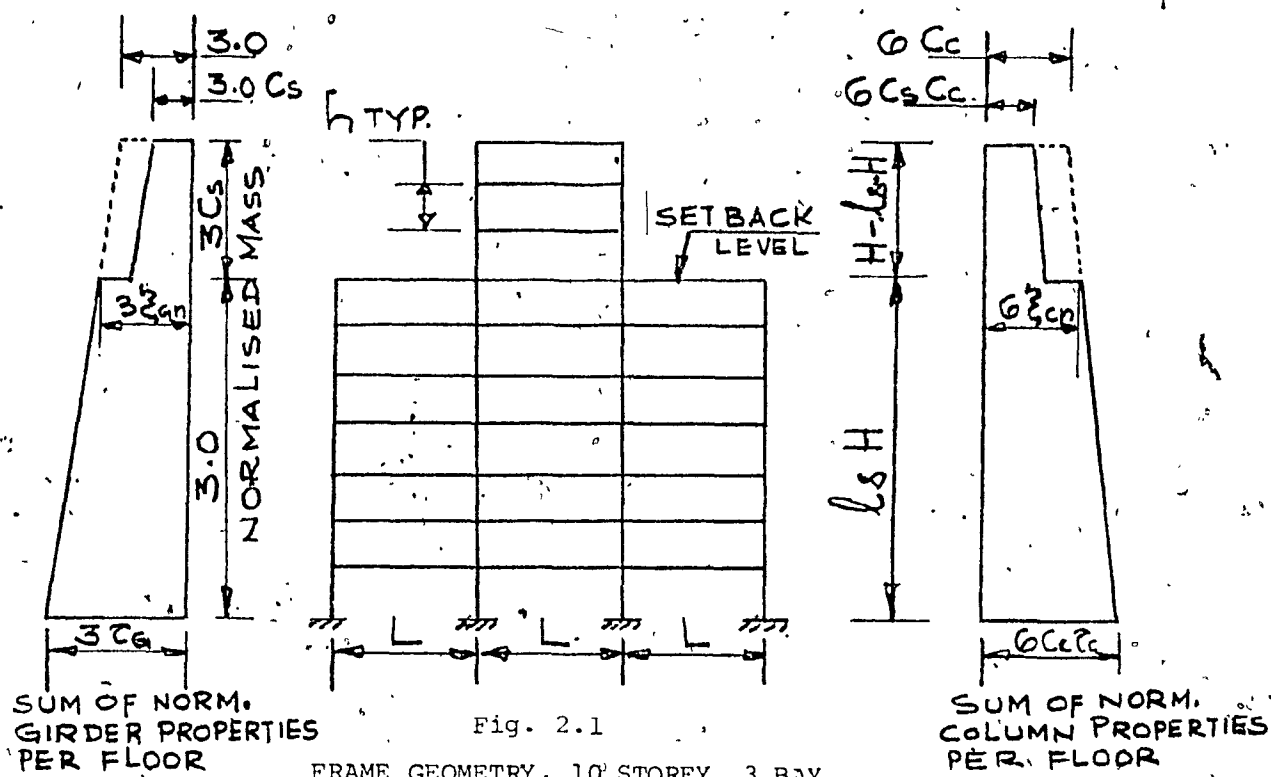
where  $EI_o$  = Reference rigidity of standard frame.

$$\tau_G = 1.5, \tau_C = 3.0, C_c = 0.7$$

$$T(\text{Funda. Period}) = 2.13 \text{ secs.}$$

$$n = \text{storey, } N = \text{Tot. no. of storey.}$$





Level of setback 0.8, 0.6, 0.4, 0.2

Degree of setback 1.0 (uniform), 0.667, 0.333, 0.167

Normalising factors  $EI_o$ ,  $M_{yo}$ ,  $m_o$

TOTAL No. of models = 13.

Basic assumption from Ref. (9):

- (i) Lumped storey mass, attached with flexible columns, sum of column length = sum of girder length. (approx.)
- (ii) Interior columns possess double stiffness and strength of exterior columns;  $T = 2.13$  secs.
- (iii) Variation of girder and column properties is a uniform taper ( $\tau_g$  &  $\tau_c$ ), Gravity loads are constant.
- (iv) Constant yield strength and flexural rigidity per unit weight of material.

Having calculated the Girder and Column properties for a 3 bay uniform structure from Equation 1 to 4 on page 7, twelve other models are generated by varying  $C_s$  (0.667, 0.333, 0.167) and  $l_s$  (0.8, 0.6, 0.4 & 0.2).

To maintain a compatible study of the behaviour of 13 different models it is very desirable that the fundamental period  $T$  of all the models be kept constant (i.e. 2.13 seconds, as for the standard frame described on page 6). This is achieved by using a routine computer programme where first the fundamental periods of different models are calculated and then the reference flexural rigidity of the respective model is adjusted to yield  $T = 2.13$  seconds as shown.

$$\omega^2 \propto \frac{EI_o}{m} \quad \text{where } \omega = \frac{2\pi}{T}$$

$$\text{or } T \propto (EI_o)^{-1/2}$$

$$\text{or } T_o (EI_o)^{1/2} = T_1 \{ (EI_o)_1 \}^{1/2}$$

$$\text{or } \{ (EI_o)_1 \}^{1/2} \text{ reqd.} = \frac{T \text{ calculated} \times (EI_o)^{1/2}}{2.13} \quad \dots (7)$$

$$T_o = 2.13 \text{ sec. for } \underline{\text{standard frame}}$$

or uniform 3 bay structure

$$EI_o = 1.88 \times 10^{10} \text{ for } \underline{\text{standard frame}}$$

The adjusted values are shown in Table 2.2 on page 13.

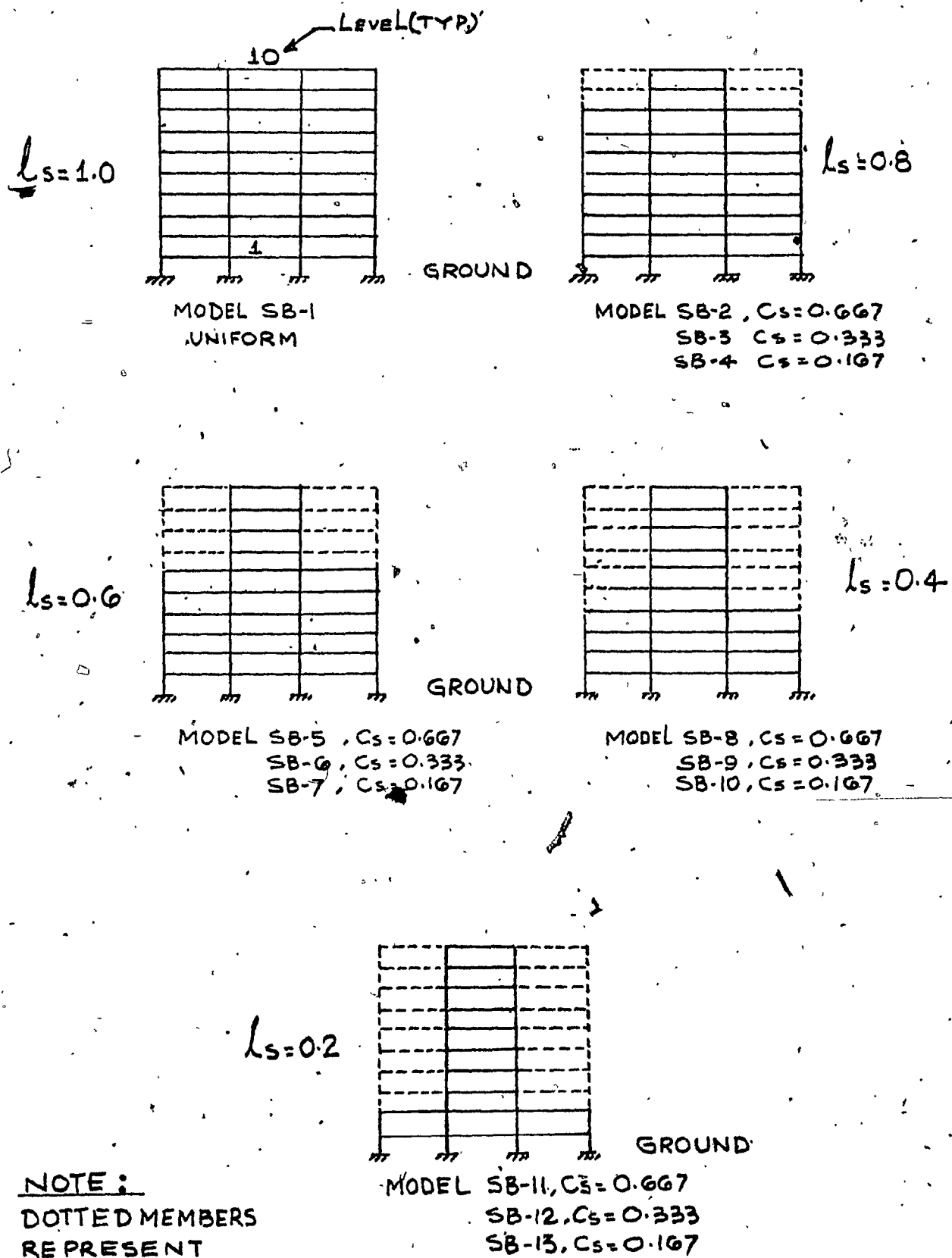


Fig. 2.2

TABLE 2.1

MODEL SB-1, MEMBER PROPERTIES (RIGIDITY &amp; YIELD MOMENT)

LEVEL	NORMALISED GIRDER PROPERTIES			NORMALISED COLUMN PROPERTIES			
	LEFT BAY	MIDDLE BAY	RIGHT BAY	EXTERIOR	INTERIOR	INTERIOR	EXTERIOR
10	1.00	1.00	1.00	0.70	1.40	1.40	0.70
9	1.05	1.05	1.05	0.85	1.71	1.71	0.85
8	1.11	1.11	1.11	1.01	2.02	2.02	1.01
7	1.17	1.17	1.17	1.17	2.33	2.33	1.17
6	1.22	1.22	1.22	1.32	2.64	2.64	1.32
5	1.27	1.27	1.27	1.47	2.95	2.95	1.47
4	1.33	1.33	1.33	1.63	3.26	3.26	1.63
3	1.38	1.38	1.38	1.78	3.57	3.57	1.78
2	1.44	1.44	1.44	1.94	3.88	3.88	1.94
1	1.50	1.50	1.50	2.10	4.20	4.20	2.10

(i) Properties shown above have been calculated from Equations 1 to 6,

page 7, with  $\frac{M_{y0}}{M_{y0}} = 1.148$ ,  $\frac{EI_0}{EI_0} = 0.951$

$$\tau_G = 1.5, \tau_C = 3.0, C_c = 0.7$$

(ii) The normalising factor  $EI_0 = 1.88 \times 10^{10}$  for uniform structure.

(iii) It can be seen that girder and column properties is a uniform taper.

(iv) It can be seen that Interior Columns possess double the values of the Exterior Columns.

(v) Properties of other models (SB-2 to SB-13) can be calculated by multiplying the above properties by degree of setback (i.e. 0.667, 0.333 or 0.167).

- (vi) Properties of Columns or Girders above the setback level will have to be further modified to conform to the sum of normalised properties, as shown on page 8.

TABLE 2.2

(ADJUSTMENT OF REFERENCE RIGIDITY)

NORMALISING FACTORS,  $EI_0$ 

MODEL	Fund. Freq. $\omega$ rad./sec. based on $EI_0 = 1.88 \times 10^{10}$	( $EI_0$ ) reqd. (for models) to yield $T = 2.13$
		( $EI_0$ ) reqd. = $\left(\frac{T_{calc.}}{2.13}\right) \times EI_0$
SB-1	$2.95 \times 10^{10}$	$1.88 \times 10^{10}$
SB-2	$3.16 \times 10^{10}$	$1.63 \times 10^{10}$
SB-3	$3.43 \times 10^{10}$	$1.38 \times 10^{10}$
SB-4	$3.59 \times 10^{10}$	$1.26 \times 10^{10}$
SB-5	$3.29 \times 10^{10}$	$1.50 \times 10^{10}$
SB-6	$3.85 \times 10^{10}$	$1.10 \times 10^{10}$
SB-7	$4.31 \times 10^{10}$	$0.88 \times 10^{10}$
SB-8	$3.25 \times 10^{10}$	$1.54 \times 10^{10}$
SB-9	$3.72 \times 10^{10}$	$1.18 \times 10^{10}$
SB-10	$4.07 \times 10^{10}$	$0.98 \times 10^{10}$
SB-11	$3.09 \times 10^{10}$	$1.70 \times 10^{10}$
SB-12	$3.29 \times 10^{10}$	$1.50 \times 10^{10}$
SB-13	$3.42 \times 10^{10}$	$1.39 \times 10^{10}$

## NOTE

- (i)  $\omega$  rad./sec. was obtained by a library programme, for each model.
- (ii) The calculated value of  $EI_0$  for each model will give equal funda. period for each model, and is used as the normalising factor for further use.
- (iii) For formulation ref. page 9.

## SECTION 3

Analytical Investigation3.1 Investigation Criteria

This section describes the various steps used to analyse the dynamic response of 13 models for the purpose of this study. Analysis is done both in elastic and inelastic range, code shears Ref. (1) are calculated and results are presented graphically and in tabular forms and, finally, comparisons are made.

The selection of SEAOC recommendations, Ref. (1), to analyse 13 models for base shear and storey shears is based in large part on the experience of engineers in Earthquake regions. The accelerated recent studies on earthquake effects on structures have led to some revisions in the design concepts that are now used and in this respect, the SEAOC (1) recommendations seem to be the most extensive and most up-to-date.

The computer programmes used for dynamic analysis are (i) DYNAL and (ii) INELAS, Ref. (11). DYNAL is basically used to calculate the fundamental period of each model and their mode shapes. The computer results (funda. period) are then used to adjust the reference flexural rigidity of different models in order to arrive at a constant value of 2.13 seconds (refer to page 9). The adjusted values are shown in Table 2.2, page 13. The adjusted values are further used as an input for INELAS (11). INELAS evaluates absolute acceleration, maximum sway, Interstorey sway, Lateral forces at different levels for multi-storey, multibay plane frame structures subjected to digitized history of ground motion (EL CENTRO QUAKE MAY 1940, N-S Component in the present study). Nonlinear, hysteretic member behaviour is represented by bilinear moment rotation relationship. The system of simultaneous differential

equations of motion is reduced to a set of algebraic equations by assuming linear variations of acceleration over small interval of time. A tridiagonalisation procedure is used to obtain the incremental time dependent solution (11).

In order that INELAS could be used to analyse frames with setback, "DUMMY MEMBERS" are introduced in the tower portion of the setback frame so that a regular frame, see Fig. 3.1, can be obtained. The dummy girders are assigned a negligibly small value of moment of inertia and full yield moment capacity (as for corresponding member of model SB-1). Dummy columns in alternate storey are assigned full rigidity and strength values (as for corresponding column of model SB-1) and other sets of dummy columns have negligible moment of inertia and full yield moment capacity. This has been done in order that yielding of dummy joints takes place in the very early part of the ground excitation so that no significant error is encountered in the dynamic behaviour of the model.

### 3.2 Parameters

In view of the fact that many unknowns do exist in complex multi-storey structures and numerous approximations must be made, no standard criteria have been set to evaluate the parameters of dynamic analysis. However, based on past experience several parameters and values are introduced.

#### Control Parameters

1. (a) Length of E.Q. record, required to determine maximum response is estimated from the variation of accelerogram intensity with time. The excitation consisting of first 15 seconds of EL CENTRO quake, May 1940, N-S component, seems adequate.



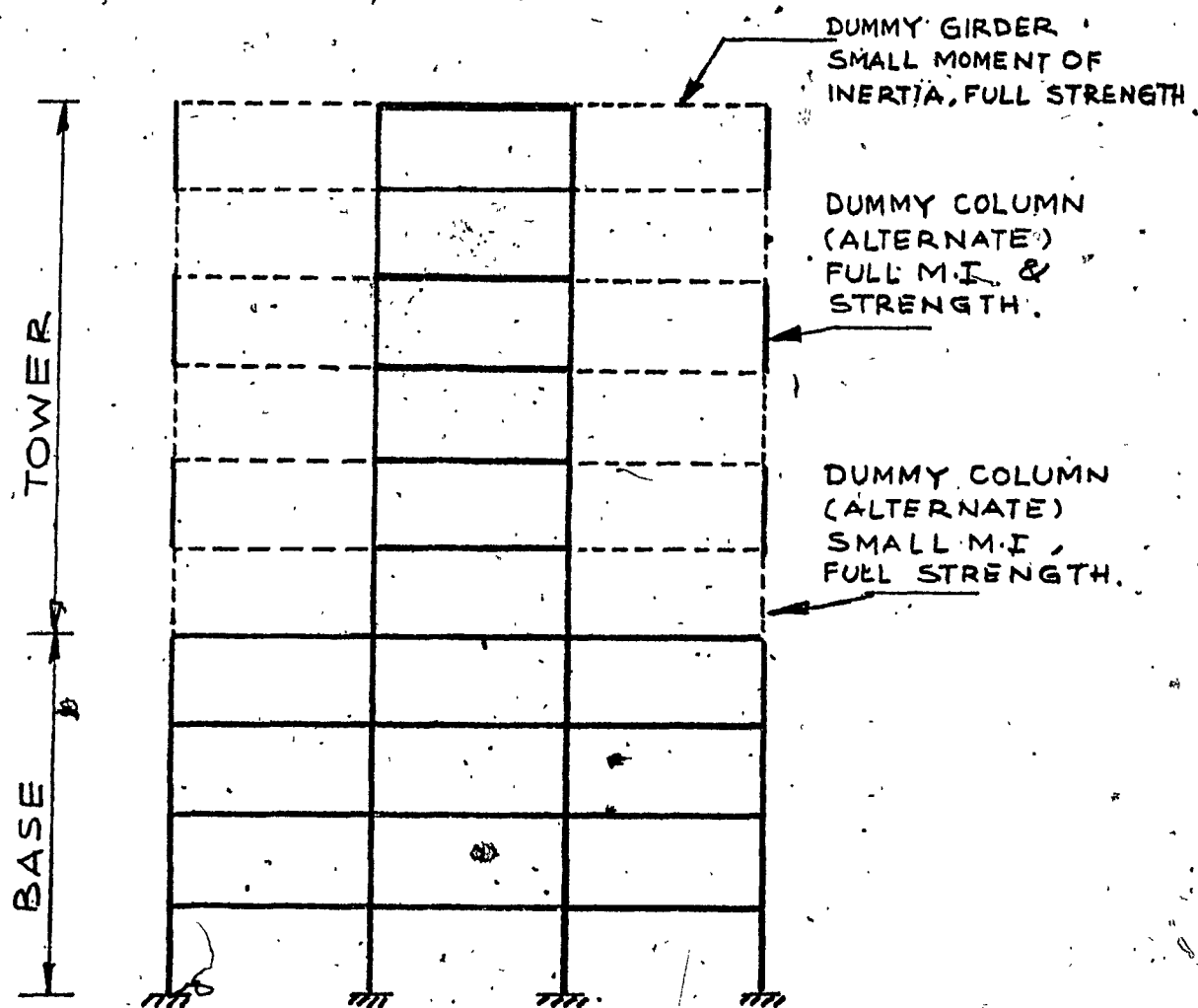


Fig. 3.1

INTRODUCTION OF DUMMY MEMBERS

(b) The time increment, which is a fraction of smallest period of vibration is chosen to be 0.005 seconds. From past experience .005 sec. has given very satisfactory results and any reduction in value requires unreasonable computer time for analysis without giving remarkably different results.

(c) The control parameter for P- $\Delta$  effect includes the dead load of the structure, since frames with weak columns and strong girders are particularly susceptible to the gravity effect.

(d) The maximum allowable error in yield level that can occur during the solution was set at 1%.

2. Intensity scale factor - the 15 seconds of EL CENTRO quake are magnified by 1.5. The combination of relatively low damping and intense excitation allows study of a level of response where the effect of yielding and participation of higher modes can be considered critical. Actual conditions to be expected would, of course, be less severe.
3. Fraction of critical viscous damping, in the form of mass proportional mechanism Ref. (9), is chosen to be 2% (usual variation between 1% to 10%); for the fundamental mode. The actual value however depends upon the type of structure and the level of response. For a modern curtain wall structure with few partitions damping ratio of 2% gives reasonable results for welded steel structures.
4. Relative damping and associated dashpot constants are assumed to be zero.
5. Bilinearity coefficient is chosen to be 0.001 and represents rate of strain hardening in plastic region, expressed as the ratio of plastic

to elastic region gradients.

INELAS (11) is versatile in nature and has been used for both elastic and inelastic analysis with use of a behaviour parameter.

### 3.3 Code Shear Response

As discussed earlier on page 14, it was decided to use SEAOC (1) recommendations to analyse the models for shear response due to ground shaking. It is also required that shear calculated from regular code provisions be adjusted depending upon geometry of setback models to approximate the actual dynamic response.

#### Lateral Force:

The regular code provisions (1) requires minimum base shear  $V$  to be determined from the equation

$$V = KCW$$

where  $W$  is the total wt. of the building and  $K$  (a coefficient) is assigned various values from 0.67 to 1.33 for buildings having different framing systems,  $K$  value depending upon estimated ductility and reserve energy capacity of structure and also taking into account the record of seismic performance of the different types of framing systems.  $C$  is a numerical coefficient for base shear and is given by

$$C = \frac{0.05}{\sqrt{T}}$$

where  $T$  is the fundamental period of vibration in seconds, in the direction considered. In absence of proper substantiated data the value of  $T$  is given by

$$T = \frac{0.05 H}{\sqrt{D}}$$

where  $H$  = Height of building above base.

$N$  = Total number of stories above grade.

$D$  = The dimension of the building in feet in a direction parallel to the applied force.

To compute storey shears the total base shear  $V$  is distributed over the height of the building in accordance with the following formula. Ref. (1),

$$F_x = \frac{V w_x h_x}{\sum w h} \quad \text{where } w_x = \text{portion of } W \text{ at level } x.$$

$$h_x = \text{Height at level } x.$$

Exception 1: one and two storey bldg. to have uniformly distributed shear.

Exception 2: when height to depth ratio of a lateral force resisting system is equal to or greater than three,  $F_t = .004 \left(\frac{H}{D}\right)^2 .15 V$  is applied at the top and the rest is distributed as above.

To utilise the above relations in a setback situation the SEAC (1) requires that

- (i) setback situations where the plan dimension of the tower in each direction is at least 75% of the plan dimension of the base, the building be considered as an uniform building without setback.
- (ii) when the setback portion is not more than 25% of the corresponding dimension of the base, the building is to be analysed as above, but fundamental period  $T$  is to be calculated on the basis of full width  $D$  of the base of the building but height  $H$  of the building be reduced to a height corresponding

to the vertical projected area on a plane parallel to the direction considered divided by base D.

- (iii) For other conditions of the setback the tower be designed as a separate building using the larger of the seismic coefficient at the base of the tower as either a separate building for its own height or as part of the overall structure. The resulting total shear force from the tower is to be applied at the top of the lower part of the building, which is considered separately for its own height.

To further modify the above requirements (i), (ii) and (iii) four adjustment procedures are outlined by the code (1), taking into consideration not only the plan dimension but also the relative heights of the tower and base portions. It is also the suggestion of the code that the structural frame be made moment resisting or braced in both tower and base portion and also the columns supporting the setback be carried straight down to the foundation.

Figure 3.2, on page 22, presents the parameters of the setback structure and lists the adjustment procedure (1) to be employed.

Procedure A requires the building to be considered as a building of full height H, with weighted average width for purpose of computing period and base shear. Procedure B requires the base to be considered a separate building of its own height with tower weight and tower base shear applied at roof level (of base). It is also required that tower base shear coefficient be at least 40% greater than that obtained on the assumption that the tower is a separate building situated on the ground. However other tower shears can be determined pro rata from this

tower base shear as for a separate building. In outlining this procedure the code recognises that base portion is predominant and tower may be considered as an appendage subject to ground motion which is equal in acceleration to that of the top of base portion.

Imaginary extension of tower through the base to the foundation level is required in Procedure C and tower is considered to be of height H and additional weight of the base not included in the so called "Extended Tower" is used as a Lean-to, having the height of the base portion. It is also required that at least 70% of all tower originated forces be provided for, within the plan limits of the extended tower. It can be seen that tower predominates.

In situations where tower and base can have considerable effect on each other, Procedure D requires the building to be analysed as (i) a building of full height H and weighted average width and calculated coefficient be increased by 20% or (ii) Procedure B, whichever governs.

To show the approach set by code (1) and as discussed in preceding paragraphs several calculations are presented below. "It is to be seen that base shear coefficient has been increased by a factor of 1.5. It can be recalled from page 13, that EL CENTRO excitation is magnified by same factor, as such the need for the increase here, in order that the study may be kept compatible." The summary of base shears is presented in Table 3.1. Table 3.2 presents the comparative values of shears calculated by code and dynamic analysis with the aid of computer.

# Adjusted Seismic Forces

(adjustment of forces as calculated from regular code provisions.)

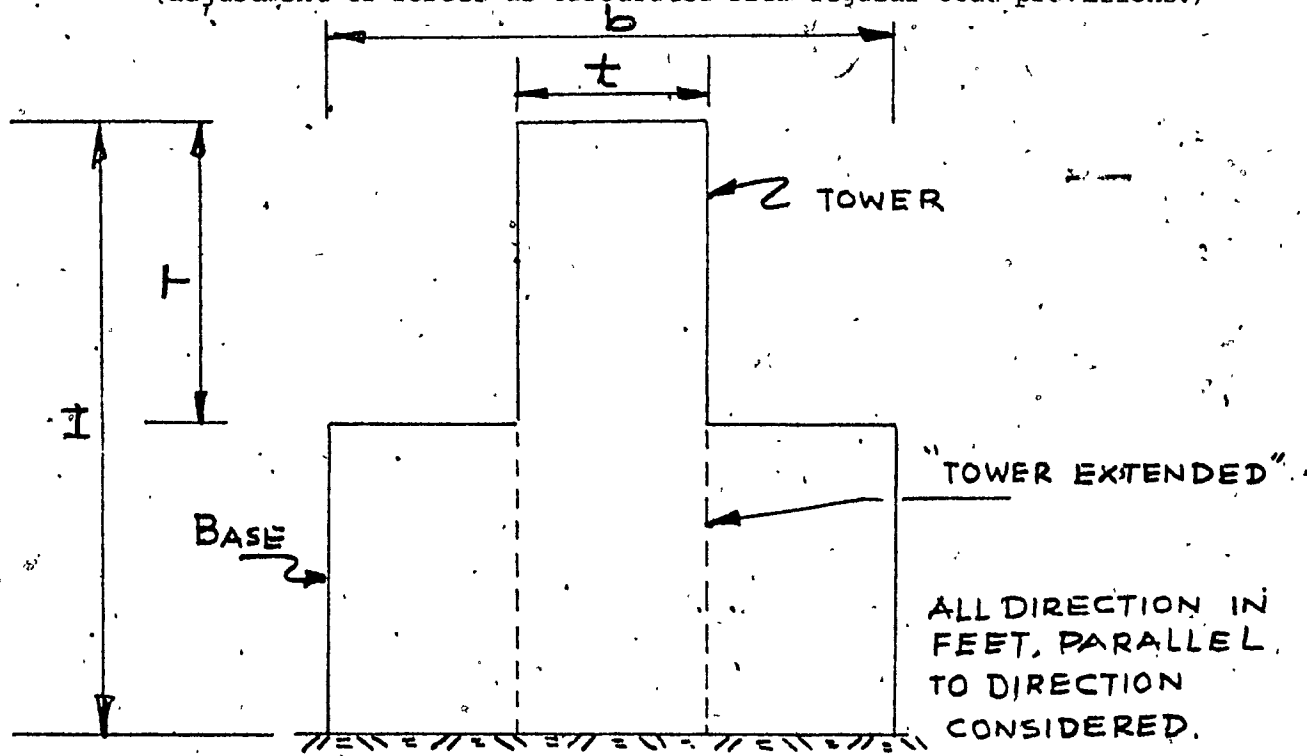


Fig. 3.2

## SET BACK STRUCTURE ELEVATION

T/H RATIO	t/b RATIO	ADJUSTMENT* PROCEDURE
1.0 or .0	0.7 0.5 0.3	A
0.2	0.7 0.5 0.3	B
0.8	0.7 0.5 0.3	C
0.6, 0.4 0.8, 0.6, 0.4	0.7 & 0.5 0.3	D

\* EXPLANATION ON PAGE NOS. 20, 21.

### 3.4 Calculation of Base and Storey Shear

#### 1) Model SB-1 (UNIFORM)

$$T = .05 \frac{H}{\sqrt{D}} = \frac{.05 \times 120}{\sqrt{60}}$$

$$= 0.776 \text{ secs.}$$

$$C = \frac{.05}{\sqrt{3T}} = \frac{.05}{\sqrt{3 \times 0.776}}$$

$$= 0.0545$$

$$KC = .0545$$

$$V = KCW = .0545 \times 1650$$

$$= 90^k \text{ TOTAL BASE SHEAR}$$

From relation, see page 19

$$F_x = \frac{V_w x h_x}{\sum w h}$$

$$F_{10} (\text{storey shear at } 10^{\text{th}} \text{ level}) = \frac{90 \times 165 \times 120}{165(120+108+96+84+72+60+48+36+24+12)}$$

$$= 16.4^k$$

Similarly  $F_9$  thru  $F_1$  are calculated.

#### 2) Model SB-2 ( $l_s = 0.8$ , $C_s = 0.667 \approx .7$ )

Using procedure B of appendix C of SEAOC (1).

Tower considered as a separate building of its own height.

Base shear coefficient (of tower):

$$\text{Base width} = 0.667 \times 60 = 40'$$

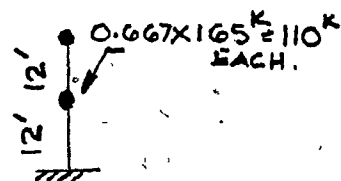
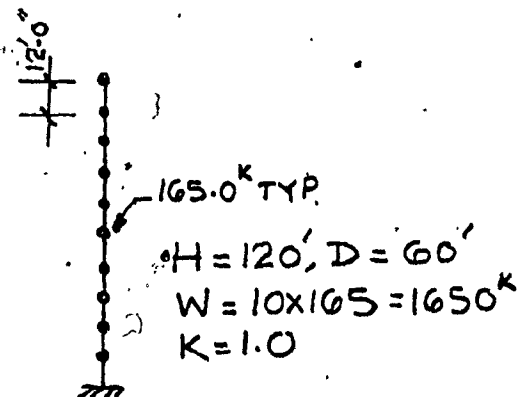
$$T = \frac{.05 \times 24}{\sqrt{40}} = .191 \text{ secs.}$$

$$C = \frac{.05}{\sqrt{3 \times .191}} = .0868$$

$$\text{Increase } C \text{ by } 40\% \quad C = 1.4 \times .0868$$

$$KC = .1215$$

$$V_{\text{Tower}} = .1215 \times 2 \times 110 = 26.7^k$$

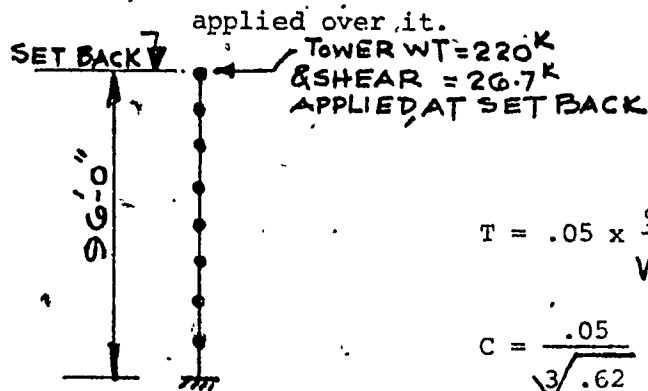




For shear distribution along storey, for two storey high tower,  
using uniform distribution

$$F_{10} = 13.35^k, F_9 = 13.35^k.$$

Base considered as a separate building with tower weight and shear



$$T = .05 \times \frac{96}{\sqrt{60}} = 0.62 \text{ sec.}$$

$$C = \frac{.05}{\sqrt[3]{.62}} = .0585$$

$$KC = .0585$$

$$\text{Total weight at setback} = 2 \times 110 + 165 = 385^k$$

$$\text{Total weight at base of base} = 385 + 7 \times 165 = 1540^k$$

portion

$$\begin{aligned} V \text{ total} &= 26.7 + .0585 \times 1540 \\ (\text{Tower} + \text{base}) &= 26.7 + 90 = 116.7^k \end{aligned}$$

Storey shear distribution

$$\begin{aligned} F_8 &= 26.7 + \frac{(90 - 26.7) \times 385 \times 96}{385 \times 96 + 165(84 + 72 + 60 + 48 + 36 + 24 + 12)} \\ &= 26.7 + \frac{63.3 \times 385 \times 96}{92,400} = 26.7 + 25.3 = 52.0^k \end{aligned}$$

$$F_7 = \frac{63.3 \times 165 \times 84}{92,400} = 9.5^k$$

Similarly  $F_6$  thru  $F_1$  can be calculated.

$$\text{SP-11 } (I_s = 0.2, C_s = 0.333)$$

Using procedure C of appendix C of SEAC (1).

Tower considered as a separate building extended through the base.

$$H = 120'$$

$$D = 0.333 \times 60 = 20'$$

$$T = \frac{.05 \times 120}{\sqrt{20}} = 1.343$$

$$C = \frac{.05}{\sqrt[3]{1.343}} = 0.0454$$

for  $\frac{H}{D} = 6 > 3$  10% of base shear is applied at top of structure or

$$F_t = 0.1V.$$

$$V(\text{tower}) = .0454 \times 10 \times (0.33 \times 165)$$

$$= 25^k$$

$$F_t = 2.5^k$$

Storey shear distribution

$$F_{10} = 2.5 + \frac{(25-2.5) \times 55 \times 120}{55(120+108+96+\dots+12)}$$

$$= 2.5 + \frac{22.5 \times 55 \times 120}{55 \times 660} = 2.5 + 4.1 = 6.6^k$$

$$F_9 = \frac{22.5 \times 55 \times 108}{55 \times 660} = 3.6^k$$

$$\text{Similarly } F_8 = 3.24^k$$

$$F_7 = 2.80^k$$

$$F_6 = 2.38^k$$

$$F_5 = 2.01^k$$

$$F_4 = 1.59^k$$

$$F_3 = 1.18^k$$

$$F_2 = 0.82^k$$

$$F_1 = 0.41^k$$

Base Additional weight not considered in above extended tower

$$= 2 \times (165 - 0.33 \times 165)$$

$$= 2 \times 110 = 220^k$$

$$T = \frac{.05 \times 24}{\sqrt{60}} = 0.156$$

$$C = \frac{.05}{\sqrt[3]{0.156}} = .095$$

$$V(\text{base}) = .095 \times 220^k = 20.9^k$$

applying at least 70% of tower originated force in the extended portion

$$V(\text{Total}) = 20.9 + 6.6 + 3.24 + 2.8 + 2.38 + 2.01 \\ + 1.59 + 1.18 + 0.7(.82 + .41) = 44.96^k$$

Revised storey shear

$$F_2 = \frac{1}{2} \times 20.9 + .7 \times .82 = 11.02^k$$

$$F_1 = \frac{1}{2} \times 20.9 + .7 \times .41 = 10.73^k$$

Model SB-4

$$C_s = 0.167 < 0.25$$

$$I_s = 0.8$$

Building treated as a uniform structure from

$$C_s = \frac{t}{b} = 0.167 \therefore t = 60 \times .167 = 10'$$

For fundamental period calculation

Vertical projected area of the setback structure = 10'x24'+96'x60'

= 6,000 Sq. ft.

Equivalent height

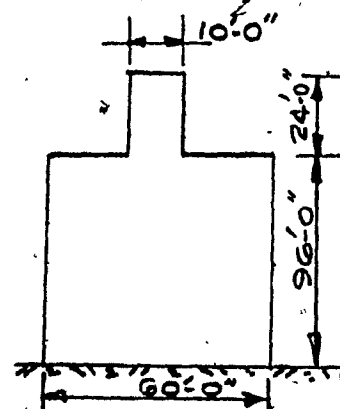
H of structure of

$$\text{base } 60' \text{ wide} = \frac{6,000}{60} = 100'$$

$$T = \frac{.50 \times 100}{\sqrt{60}} = .646$$

$$C = \frac{.05}{\sqrt[3]{.646}} = .0578$$

$$KC = .0578$$



$$\frac{H}{D} < 3 \quad \text{use } F_t = 0.$$

$$\begin{aligned} \text{Total storey weights} &= 2 \times .167 \times 165 + 8 \times 165 \\ &= 1375^k, \end{aligned}$$

$$V = .0578 \times 1375 = 79.5^k$$

Storey shear distribution

$$\begin{aligned} F_{10} &= \frac{79.5 \times 27.5 \times 120}{27.5 \times (120 + 108) + 165(96 + 84 + \dots + 12)} \\ &= \frac{79.5 \times 27.5 \times 120}{77,460} = 3.40^k \end{aligned}$$

Similarly  $F_9$  thru  $F_1$ .

TABLE 3.1

(SUMMARY OF SHEARS)  
CALCULATED FROM CODE CRITERIA

MODEL	PROCEDURE	STOREY SHEAR IN KIPS										TOTAL BASE SHEAR	TOTAL STOREY WEIGHT
		10th	9th	8th	7th	6th	5th	4th	3rd	2nd	1st		
SB-1	A	24.6	22.2	19.7	17.4	14.7	12.3	9.9	7.4	5.0	2.0	135.6	1650
SB-2	B	20.1	20.1	18.0	14.3	12.2	10.0	8.1	6.2	4.4	2.4	175.8	1540
SB-5	B, D	25.5	19.2	12.9	6.5	107.8	10.1	8.1	6.00	4.1	3.0	203.2	1430
SB-8	B	23.9	20.0	16.1	12.0	8.0	4.1	132.3	7.4	5.0	3.6	232.4	1320
SB-11	C	22.2	12.5	11.1	9.6	8.3	6.9	5.4	4.2	9.8	8.9	98.9	1210
SB-3	B	9.0	9.0	52.4	18.3	15.6	13.1	10.4	7.8	5.3	2.6	143.5	1430
SB-6	B, D	11.4	8.4	5.6	2.9	71.4	15.2	12.3	9.3	6.2	3.2	145.9	1210
SB-9	B	13.4	8.1	6.5	4.8	3.2	1.6	86.5	12.0	8.1	4.1	148.3	980
SB-12	C	9.9	5.4	4.8	4.2	3.6	3.0	2.4	1.8	16.5	16.1	67.7	770
SB-4	A	5.1	4.7	24.5	21.4	18.3	15.3	12.3	9.2	6.1	3.0	119.9	1375
SB-7	A	6.5	5.7	5.3	4.5	22.4	19.4	15.5	11.5	7.8	3.9	102.6	1100
SB-10	A	8.1	7.4	6.5	5.7	5.0	4.1	19.4	14.6	9.8	4.8	85.4	825
SB-13	A	9.3	8.4	7.5	6.5	5.6	4.7	3.8	2.7	11.1	5.6	65.2	550

NOTE: All shear values noted above have been increased by 1.5, as calculated from Code Criteria shown on pages 23-27. This has been done to account for earthquake intensity factor of 1.5 which has been used for inelastic and elastic analysis.

STOREY AND BASE SHEAR  
CUMMULATIVE, KIPS

MODEL	APPROACH	10 <sup>th</sup>	9 <sup>th</sup>	8 <sup>th</sup>	7 <sup>th</sup>	6 <sup>th</sup>	5 <sup>th</sup>	4 <sup>th</sup>	3 <sup>rd</sup>	2 <sup>nd</sup>	1 <sup>st</sup> & BASE	RATIO OF BASE SHEAR
SB-1	Code	24.6	46.8	66.5	83.9	98.6	110.9	120.8	128.2	133.2	135.6	2.8
	Inelas.	152.4	178.8	186.3	212.6	239.9	257.2	257.3	303.6	317.4	379.4	1.84
	Elas.	280.6	383.4	449.2	463.1	569.4	492.3	567.1	613.3	653.0	698.1	
SB-2	Code	20.1	40.2	118.2	132.5	144.7	154.7	162.8	169.0	173.4	175.8	1.97
	Inelas.	221.2	159.9	202.1	206.1	235.4	275.0	250.2	297.0	307.3	345.8	2.08
	Elas.	229.2	328.2	325.1	374.6	474.3	530.4	509.9	547.0	698.8	717.1	
SB-5	Code	25.5	44.7	57.6	64.1	171.9	182.0	190.1	196.1	200.2	203.2	1.69
	Inelas.	122.6	141.5	149.0	182.2	212.7	252.2	247.8	276.5	329.5	342.2	2.25
	Elas.	268.4	339.7	385.3	360.1	493.7	489.8	511.1	524.9	669.2	773.2	
SB-8	Code	23.9	43.9	60.0	72.0	80.0	84.1	216.4	223.8	228.8	232.4	1.59
	Inelas.	129.7	153.6	141.7	150.0	165.8	211.7	227.7	269.9	327.4	354.7	1.74
	Elas.	223.7	317.2	332.1	320.3	357.5	414.3	409.5	474.2	542.2	617.9	
SB-11	Code	22.2	34.7	45.8	55.4	63.7	70.6	76.0	80.2	90.0	98.9	3.31
	Inelas.	118.6	152.6	156.5	151.2	168.2	193.2	179.7	237.9	284.8	327.0	1.68
	Elas.	169.6	228.8	266.5	358.1	351.2	352.2	345.7	431.2	482.9	548.6	

COMPARATIVE SUMMARY

TABLE 3.2

STOREY AND BASE SHEAR  
CUMULATIVE, KIPS

RATIO OF  
BASE  
SHEAR

1<sup>st</sup> &  
BASE

2<sup>nd</sup>

3<sup>rd</sup>

4<sup>th</sup>

5<sup>th</sup>

6<sup>th</sup>

7<sup>th</sup>

8<sup>th</sup>

9<sup>th</sup>

10<sup>th</sup>

MODEL APPROACH

Code

Inelas.

Elas.

Code

Inelas.

Elas.

Code

Inelas.

Elas.

Code

Inelas.

Elas.

Code

Inelas.

Elas.

2.8

1.84

1.43

2.55

1.43

2.4

1.70

2.51

1.86

4.03

2.01

COMPARATIVE SUMMARY

TABLE 3.2

# STOREY AND BASE SHEAR

## CUMULATIVE, KIPS

MODEL	APPROACH	10 <sup>th</sup>	9 <sup>th</sup>	8 <sup>th</sup>	7 <sup>th</sup>	6 <sup>th</sup>	5 <sup>th</sup>	4 <sup>th</sup>	3 <sup>rd</sup>	2 <sup>nd</sup>	1 <sup>st</sup> & BASE	RATIO OF BASE SHEAR
SB-1	Code	24.6	46.8	66.5	83.9	98.6	110.9	120.8	128.2	133.2	135.6	1.28
	Inelas.	152.4	178.8	186.3	212.6	239.9	257.2	257.3	303.6	317.4	379.4	1.84
	Elas.	280.6	383.4	449.2	463.1	569.4	492.3	567.1	613.3	653.0	698.1	
SB-4	Code	5.1	9.8	34.3	55.7	74.0	89.3	101.6	110.8	116.9	119.9	3.04
	Inelas.	33.5	41.2	141.1	166.1	210.1	225.6	281.7	273.8	303.9	363.9	1.67
	Elas.	97.1	126.0	239.4	299.5	386.5	439.4	449.4	477.3	521.5	607.2	
SB-7	Code	6.5	12.2	17.5	22.0	44.4	63.8	79.3	90.9	98.7	102.6	3.65
	Inelas.	33.5	41.3	41.8	51.8	194.4	231.3	221.1	250.8	310.6	373.7	1.51
	Elas.	64.8	90.6	90.4	127.6	293.3	386.6	334.2	356.2	462.5	564.6	
SB-10	Code	8.1	15.5	22.0	27.7	32.7	36.8		70.8	80.6	85.4	3.48
	Inelas.	33.7	38.7	48.6	44.1	47.3	58.3	184.1	248.5	271.9	296.3	1.64
	Elas.	61.5	85.0	95.0	86.2	99.9	104.8	261.6	337.3	404.4	485.3	
SB-13	Code	9.3	17.7	25.2	31.7	37.3	42.0	45.8	48.5	59.6	65.2	5.1
	Inelas.	33.6	40.8	42.1	48.4	52.7	53.4	55.9	62.1	225.6	332.5	2.04
	Elas.	116.8	145.1	90.1	128.6	155.4	134.0	121.9	148.8	474.0	677.0	

## COMPARATIVE SUMMARY

TABLE 3.2



## SECTION 4

## RESULTS &amp; DISCUSSION

A discussion on the results of this study are outlined in this section. Although the determination of the shear force resulting from ground shaking is the most important factor for the structural design, other response parameters are also compared.

However, the code criteria could only be used for the comparison of shear response (as it does not deal with other important response). Exact dynamic analysis based on inelastic and elastic behaviour has been used to compare other response parameters.

#### 4.1 Fundamental Period

The values of fundamental period shown in Fig. 4.1 and 4.2 have been obtained on the basis of constant flexural rigidity for each model (Ref. page 13). From the variation of fundamental period against level of setback, Fig. 4.1 and degree of setback Fig. 4.2, the following points are noted.

(i) Period for the first mode gradually decreases in value as the level of setback decreases from  $l_s = 1.0$  (uniform) with minimum values for  $l_s = 0.6$  or  $0.4$ . The reason for this behaviour can be attributed to the fact that models with  $l_s = 0.8$  and  $0.2$  closely resemble a uniform structure and effect of tower (for  $l_s = 0.8$ ) or effect of base (for  $l_s = 0.2$ ) is very little on the overall structure. In models with  $l_s = 0.6$  and  $0.4$  both portions of the structure have considerable effect on each other.

However, the variation of period for second mode gradually increases

for setbacks at about mid height, values for uniform and models with  $l_s = 0.8$  and  $0.2$  closely resembling each other.

The curve for variation of period (1st and 2nd) with degree of setback (Fig. 4.2) is approximately parallel for each subgroup of models (of similar  $l_s$ ). The values of 1st period decrease almost linearly with decreasing  $C_s$  while values for 2nd period are nearly independent of  $C_s$ .

It can be noted from Fig. 4.1 and 4.2 and this discussion that it is the level of setback which determines whether the structure will behave closely to a uniform structure or otherwise, not the degree of setback.

#### 4.2 Mode Shapes

The first two mode shapes of models are shown in Figs. 4.3.A, 4.3.B and 4.3.C for each subgroup of models (having equal degree of setback). The response of uniform structure is also plotted to show the effect of setback. The model displacements have been normalised by fixing the displacement at the top of the structure equal to unity, to allow ready comparison of shapes.

1st MODE: Models with setback have generally less displacements than comparable uniform structure at mid levels. This behaviour is true for each subgroup of models of similar  $C_s$ . The models with  $l_s = 0.6$ ,  $0.4$  and  $0.2$  show a reduction in displacement as compared to uniform model at mid levels as  $C_s$  is decreased. The greatest reduction being noticed for  $l_s = 0.6$  and  $0.4$ . An abrupt change in shape is noticed at level of setback (irrespective of  $C_s$ ) which may result in whiplashing effect for certain structures.

2nd MODE: Shapes for  $l_s = 0.2$  and  $0.4$  are found to resemble comparable uniform structure while models with  $l_s = 0.8$  and  $0.6$  show the

largest setback effect. The displacement at mid levels of models with  $l_s = 0.8$  and  $0.6$  become smaller as the degree of setback is reduced.

This may imply that contributions of higher modes to the shear response of models with  $l_s = 0.2$  and  $0.4$  is comparable to a uniform structure and is not greatly affected by change in degree of setback. The shear contribution due to second mode however becomes smaller due to decrease in  $C_s$  (degree of setback).

#### 4.3 Total Storey Shear

Considerable variations in the shear response of models is noticed by employing three different methods of analysis (namely code criteria, elastic and inelastic analysis). As also stated earlier in Section 1, it is found that inelastic analysis gives shear values between those of code and elastic analyses. The comparison of base shear and cumulative shear values at storey levels was presented in Table 3.2 of Section 3. The tabular values are presented graphically in Figs. 4.4.A, 4.4.B and 4.4.C for each subgroup of models (of equal  $C_s$ ) and the following observations are made:

(i) Distribution of shear is strongly influenced by the setback geometry.

(ii) Marked and sudden change in shear occurs at the setback level. The largest change in storey shear occurs, according to code, for setback where  $l_s = 0.4$  and  $C_s = 0.667$ , Fig. 4.4.A. The code criteria in general predict larger discontinuities in the shear envelope for models with large  $C_s$  and setback at about mid height. The effect of the setback decreases with decreasing degree of setback.

For elastic and inelastic analysis, however, structures with tall tower portions and large setbacks have shear envelopes that deviate widely from that of the uniform case (Ref. Fig. 4.4.C). Higher values for  $C_s$  and  $l_s$  tend, as expected, to lead to the shear envelope of the uniform case.

(iii) No consistent pattern of comparison in the values of base shear is noticed, as obtained from three approaches. The ratio of inelastic to code base shear is found to be in the range of 1.5 to 5.0. The ratios can be grouped as follows:

for uniform model	Ratio	3.0
models with $l_s = 0.8$	Range of Ratio	2.0 to 3.0
models with $l_s = 0.6$ & $0.4$	Range of Ratio	1.5 to 3.5
models with $l_s = 0.2$	Range of Ratio	3.0 to 5.0

The models with  $C_s = 0.667$ , yield the lowest ratio. The ratio of elastic to inelastic base shear is in the neighbourhood of 2.

The above underestimation of shears for even inelastic behaviour by the code suggest that the 2% viscous damping of this study needs to be re-examined. It is expected that use of 5 - 10% would yield better agreement between inelastic and code forces.

#### 4.4 Absolute Acceleration

The variation of absolute floor acceleration with floor levels is shown in Figs. 4.5.A, 4.5.B and 4.5.C and the following observations are noted.

(i) There is a sudden increase in floor acceleration of setback levels, irrespective of  $C_s$  and  $l_s$  values.

(ii) Maximum value of absolute acceleration is found to occur in models with setback level below mid height, models with  $l_s = 0.2$  generally yielding highest acceleration irrespective of  $C_s$ . However, the maximum acceleration generally occurs in the top few floors of the setback structure.

(iii) Magnitude of maximum acceleration increases as the degree of setback decreases.

(iv) Magnitude of maximum absolute acceleration is generally greater in elastic case than inelastic case irrespective of setback geometry.

(v) The effect of yielding for the case of largest absolute acceleration encountered was to reduce the peak value as shown.

for $l_s = 0.2$ , $C_s = 0.167$ :	4.3G	(elastic case)
	Ref.	(inelastic case)
	2.1G	(inelastic case)

#### 4.5 Interstorey Sway

The variation of interstorey sway with storey levels is shown in Fig. 4.6.A, 4.6.B and 4.6.C., and the following observations are presented.

(i) Sudden increases in inelastic interstorey sway at setback levels are observed. The increase is larger for models with smaller  $C_s$  and smaller tower (Ref. Fig. 4.6.C); maximum increase appears to occur in model with smallest tower ( $l_s = 0.8$  and  $C_s = 0.167$ ). However, for elastic analysis, such increased tower response is obtained for models with taller towers.

(ii) Maximum magnitude of sway is found at  $l_s = 0.6$ ,  $C_s = 0.167$ , for both elastic and inelastic case, (Ref. Fig. 4.6.C) occurring at upper storey levels.

#### 4.6 Total Sway

From the variation of sway along floor levels, as shown in Figs. 4.7.A, 4.7.B, 4.7.C the following observations are made.

- (i) The sway curves are similar to the first mode shape irrespective of setback geometry.
- (ii) Generally no considerable difference is observed in the magnitude of sway due to variation in level of setback except at few top stories, irrespective of degree of setback or vice versa.
- (iii) Generally there are no abrupt changes in the magnitude of sway at setback level.
- (iv) Elastic sway is generally slightly greater than the inelastic sway.

#### 4.7 Column Ductility Factors

The envelopes of maximum column ductility factors are presented in Figs. 4.8.A, 4.8.B and 4.8.C and following behaviour is observed.

- (i) Large column ductility factors occur in the tower portion of setback structures.
- (ii) The maximum increase in ductility factor at setback level is found to occur for  $l_s = 0.8$  and  $C_s = 0.167$  (Fig. 4.8.C).
- (iii) Values are higher in inelastic case than the corresponding factors for elastic behaviour.

#### 4.8 Girder Ductility Factors

From the envelopes of maximum girder ductility factors, presented in Figs. 4.9.A, 4.9.B, 4.9.C, the following behaviour is observed.

(i) Girders in top and bottom floors exhibit roughly similar ductility factors.

(ii) Maximum yielding is observed for most cases in intermediate floors (between 4th and 8th) irrespective of model geometry.

(iii) Abrupt increases in values occur at setback levels, maximum increase occurring in model with  $l_s = 0.8$ ,  $C_s = 0.167$ , Fig. 4.9.C.

(iv) No definite pattern is observed in the change of girder ductility factor by varying the degree of setback; however, they generally tend to become larger as  $C_s$  is reduced (only at few locations and more significantly in elastic response).

(v) No significant difference is observed in the maximum values while comparing elastic and inelastic response; however, values differ significantly when considering a particular floor.

#### 4.9 Graphical Representation of Results

Refer to Figs. 4.1 to 4.8.C, on pages 40 to 83. Response plots represent the envelopes of maximum absolute values incurred at different floor levels.

#### CONCLUSION

The data presented in this report applies only to a particular set of model structures subjected to the 1940 EL CENTRO NS earthquake record, and as such, has limited application to other structures. In particular, the degree and model for viscous damping need to be examined further.

In order to make possible general rules of behaviour, and especially to evaluate suitable code guidelines that allow for inelastic behaviour of setback structures, additional studies are required using an ensemble of earthquake motions. At the same time, response data over the range of frequencies of interest should be obtained. Thus this study should only be considered as an initial attempt of the problem as a whole.



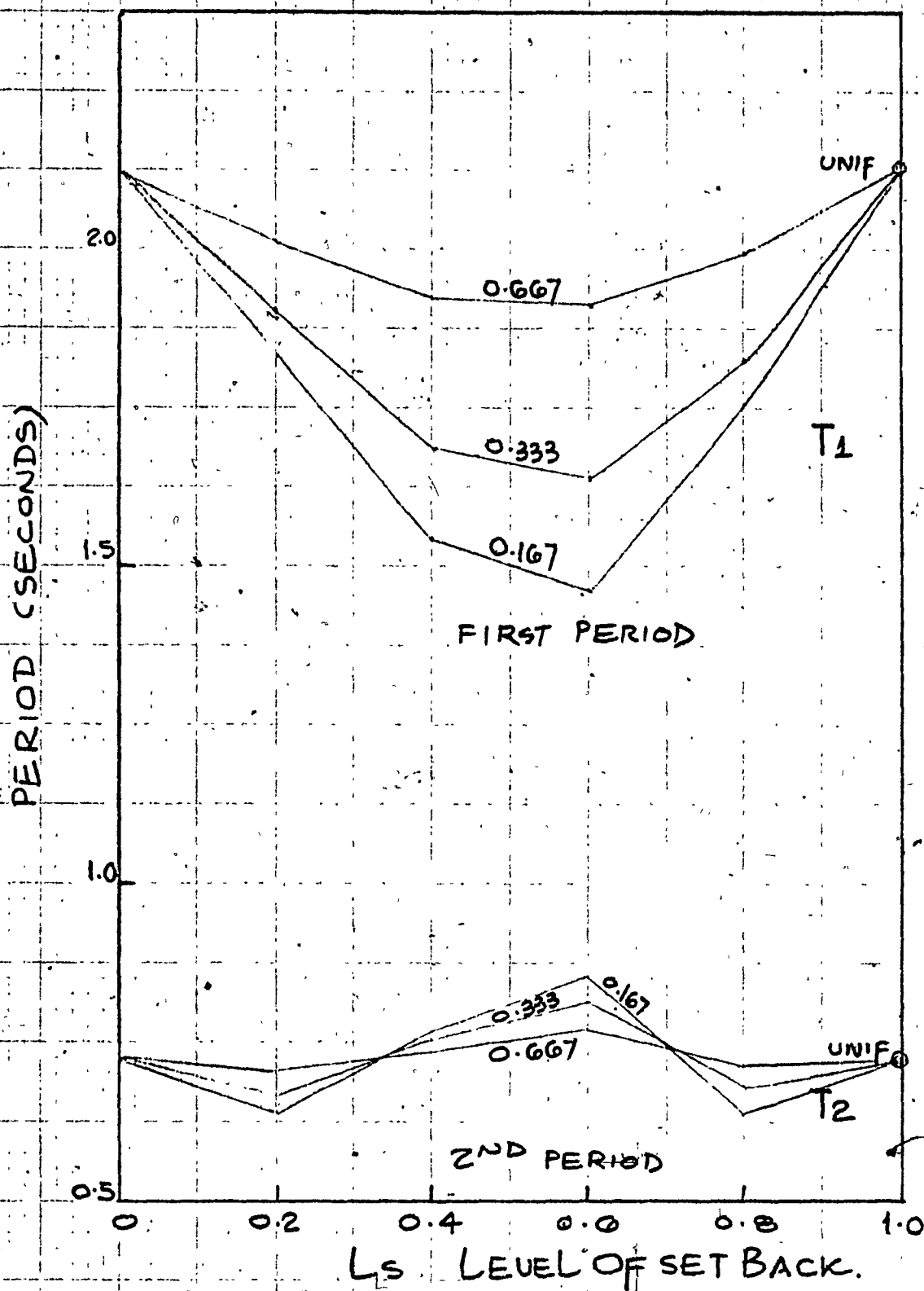


FIG. 4.1

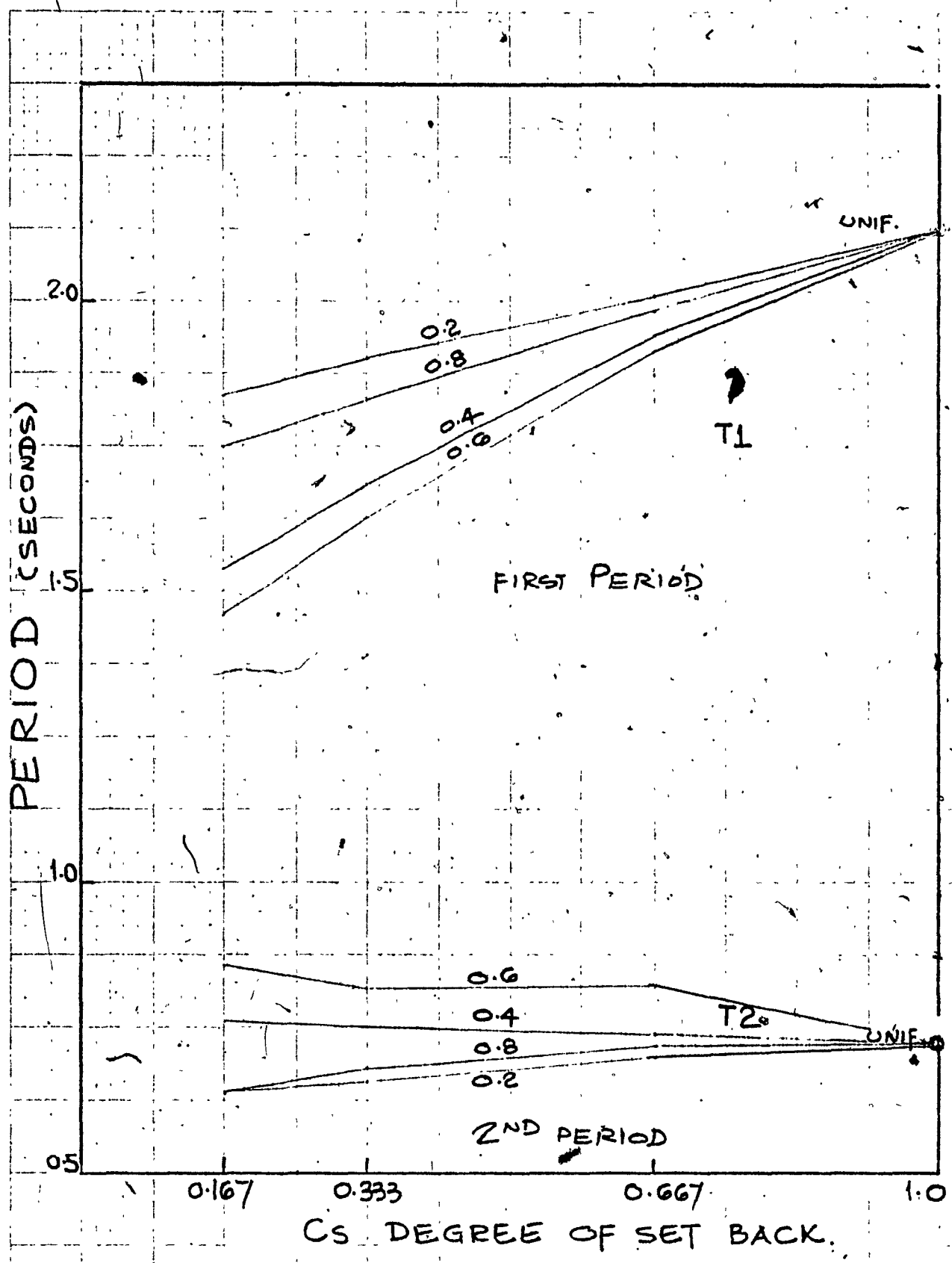


FIG. 4.2

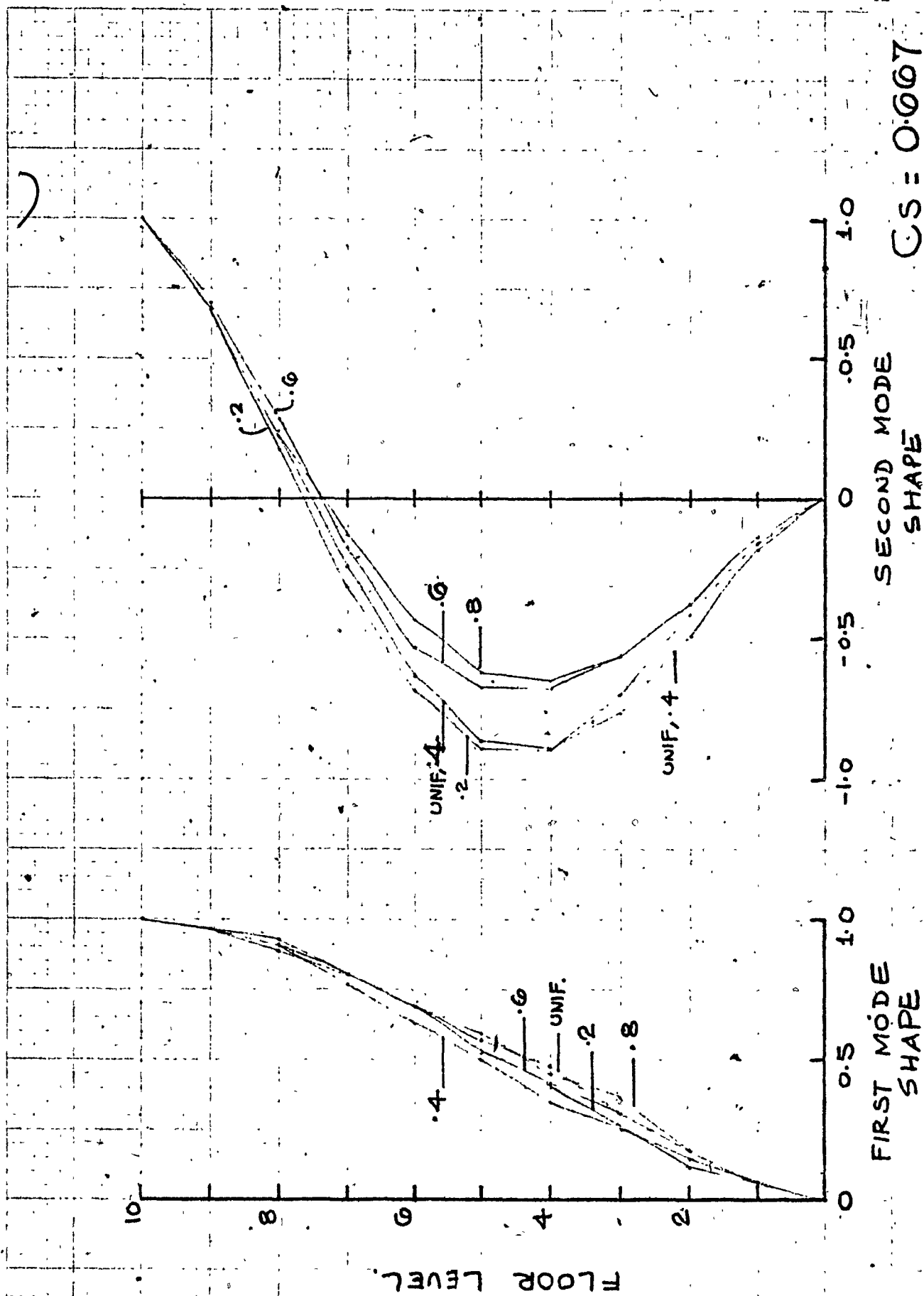
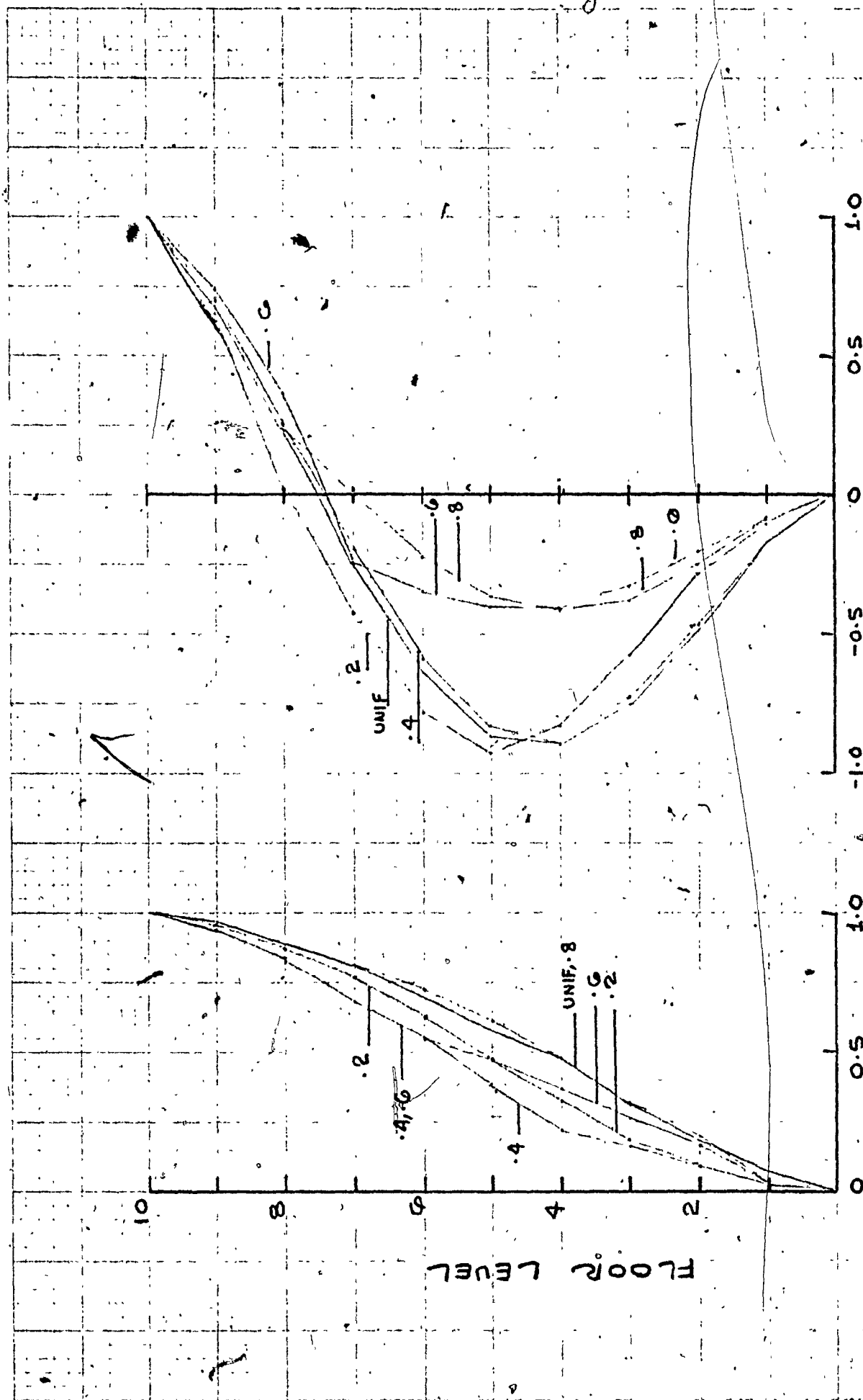


FIG. 4.3.A

CS = 0.007

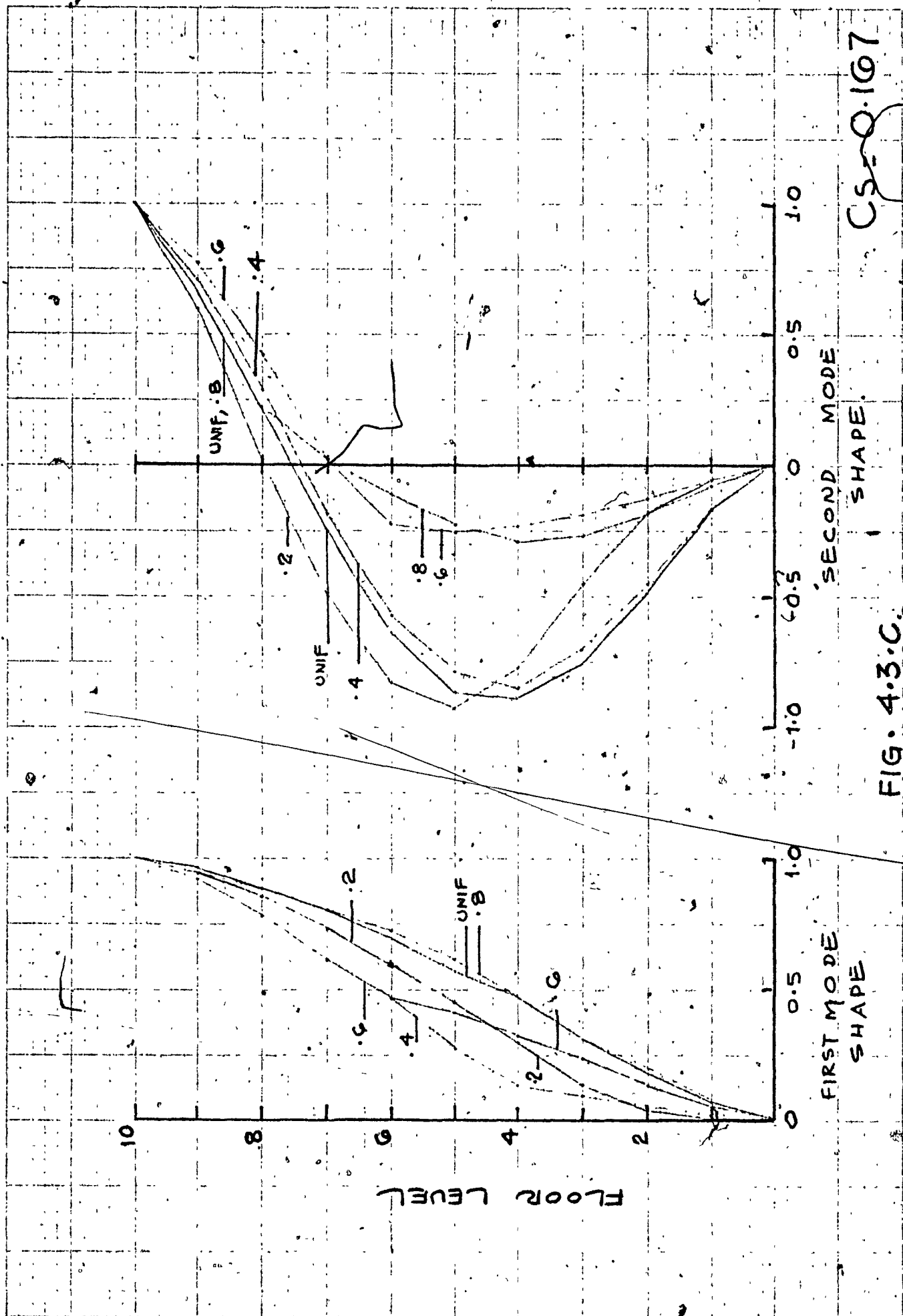


$C_s = 0.333$

SECOND MODE  
SHAPE.

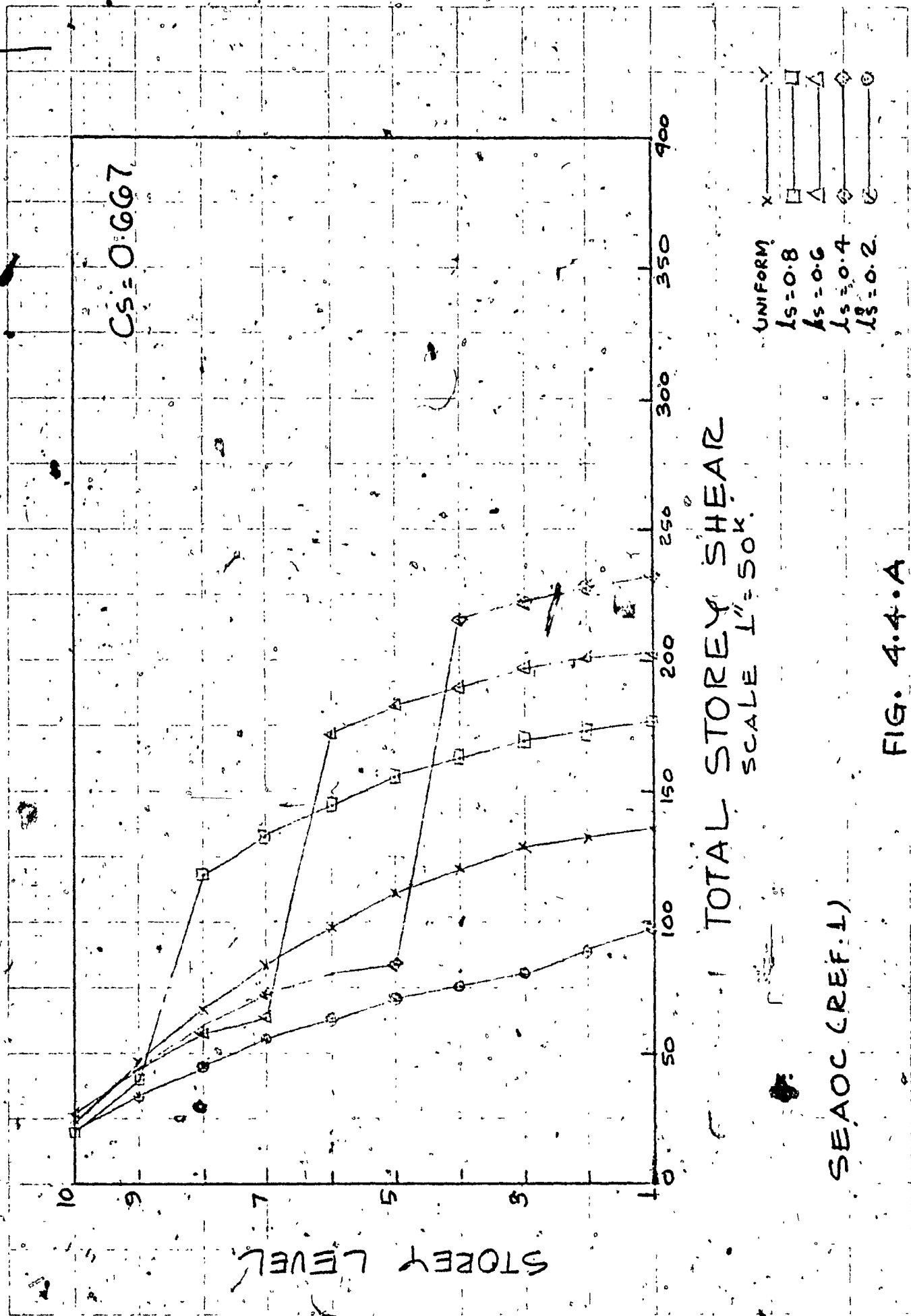
FIRST MODE  
SHAPE

FIG. 4.3.B



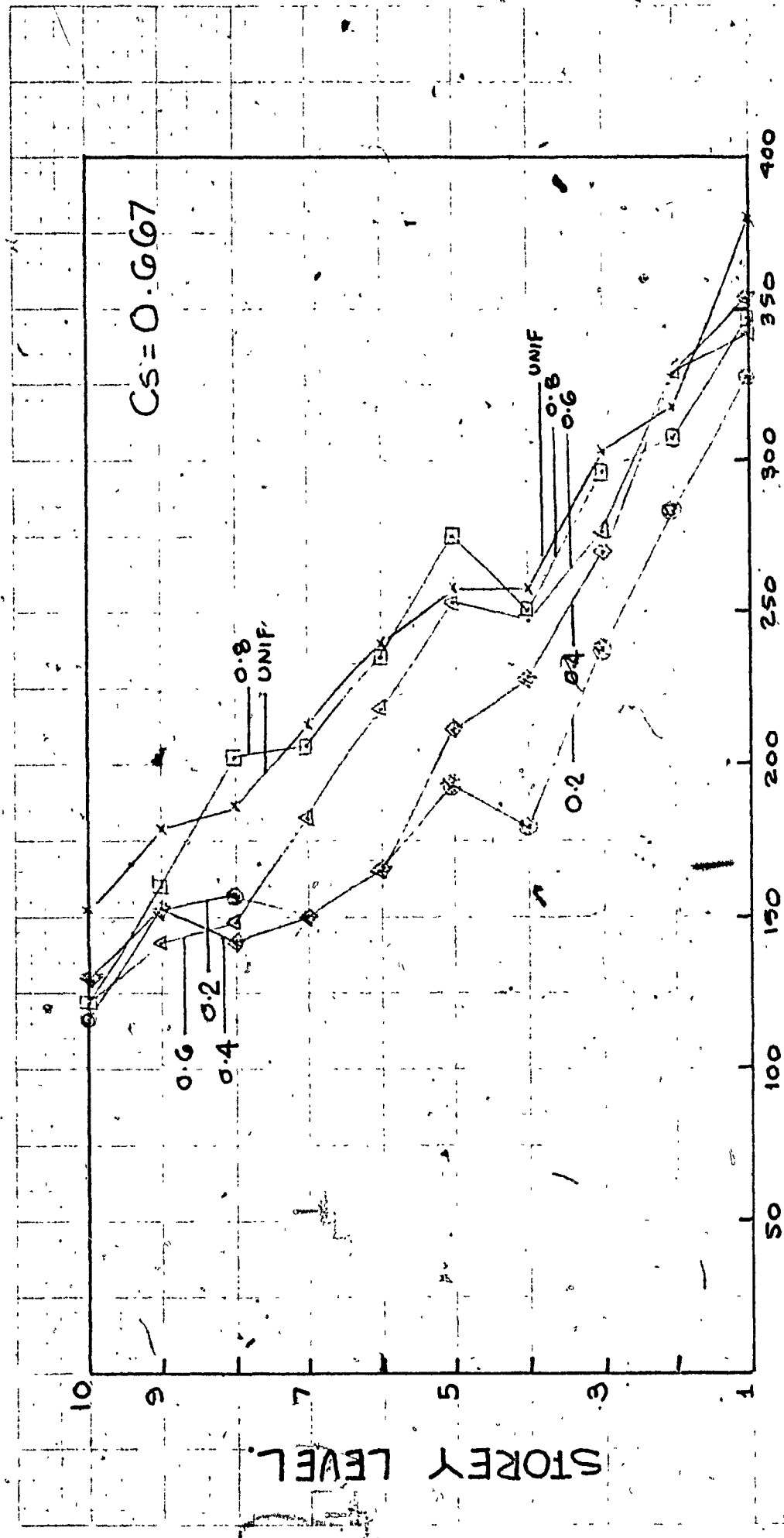
CS-0.107

FIG. 4.3.C



SEAOC (REF. 1)

FIG. 4.4.A



- UNIFORM
- $\lambda_s = 0.8$
- $\lambda_s = 0.6$
- $\lambda_s = 0.4$
- $\lambda_s = 0.2$

INELASTIC RESPONSE.

FIG 4.4.A

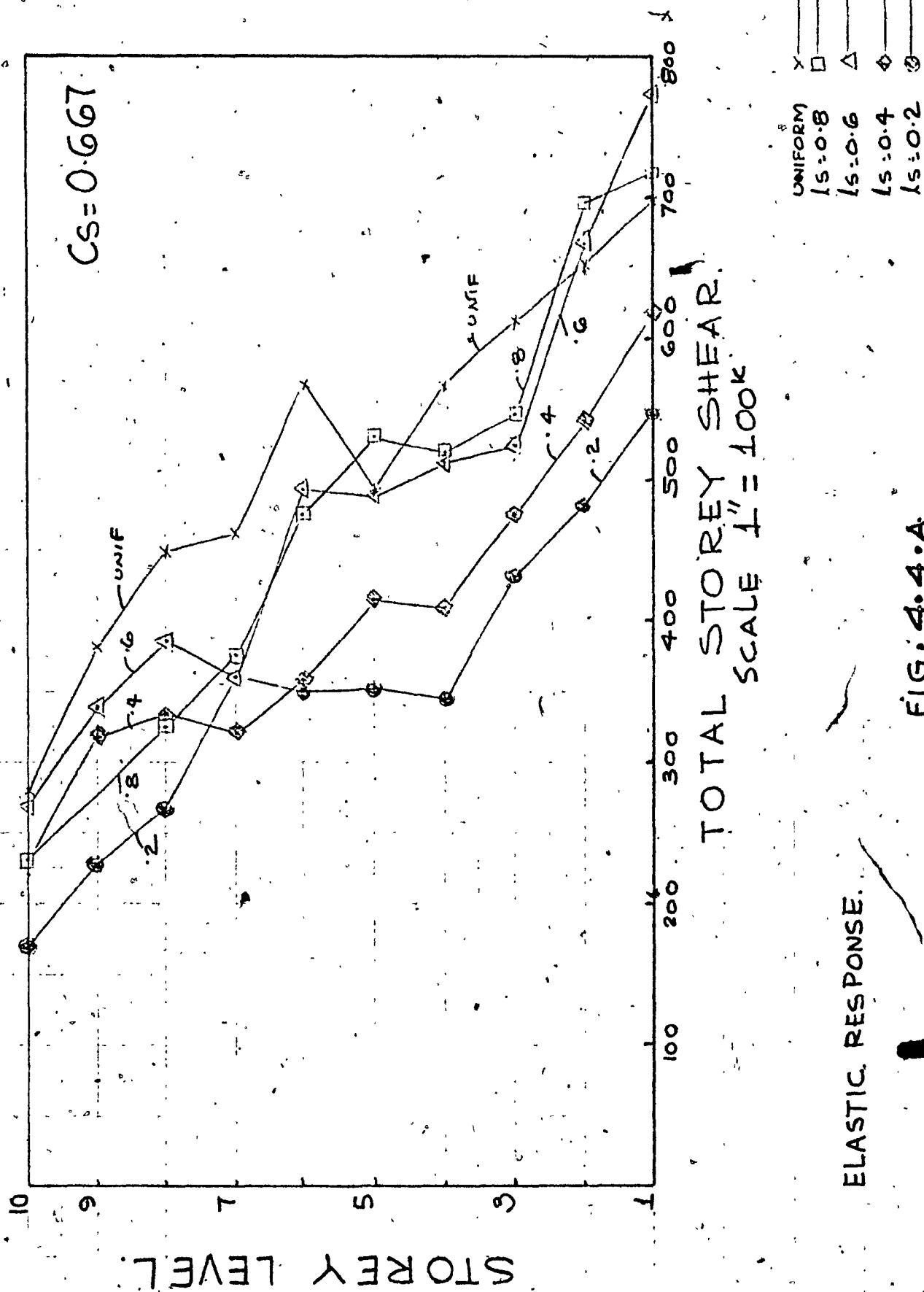
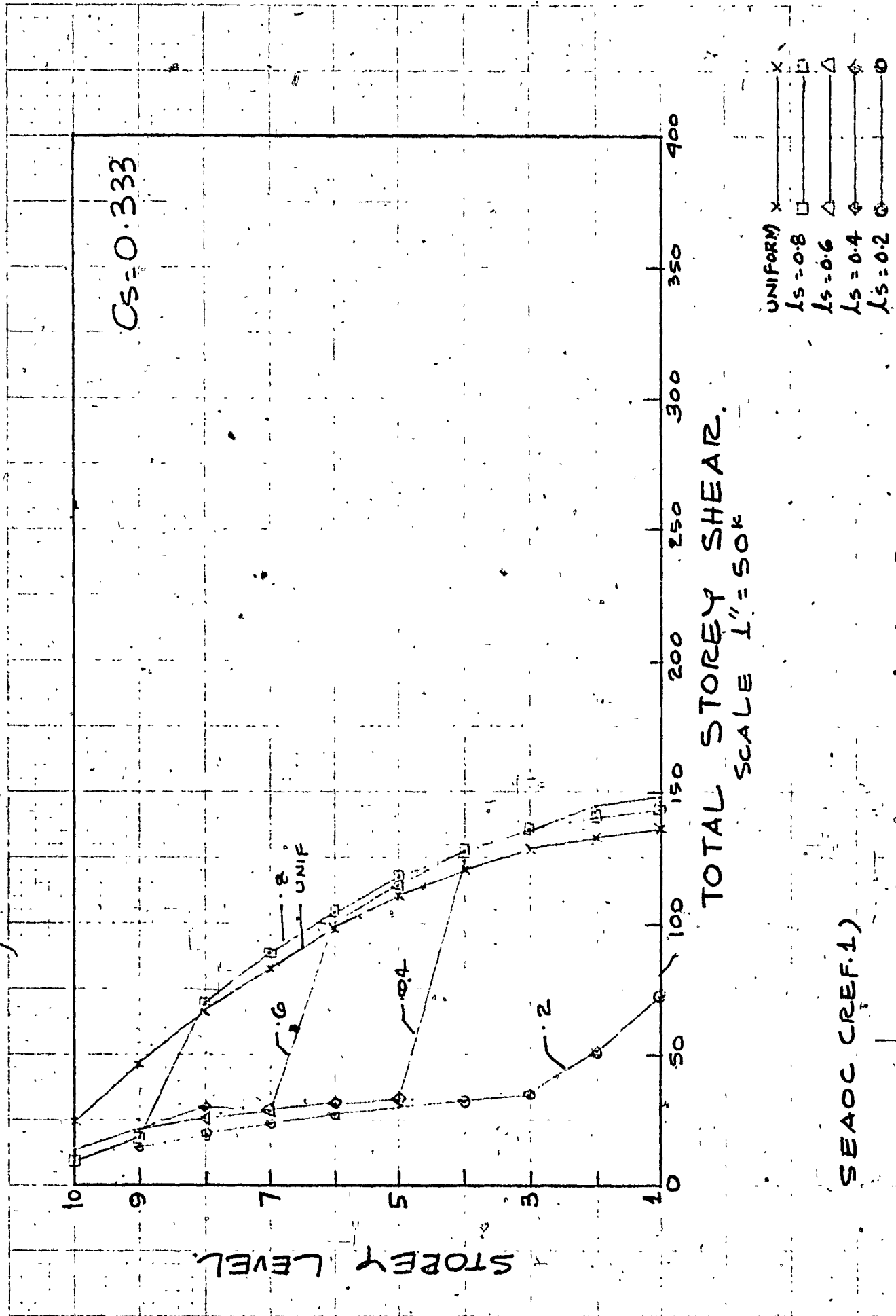


FIG. 4.4.A





SEAOCC (REF.1)

FIG. 4.4.B

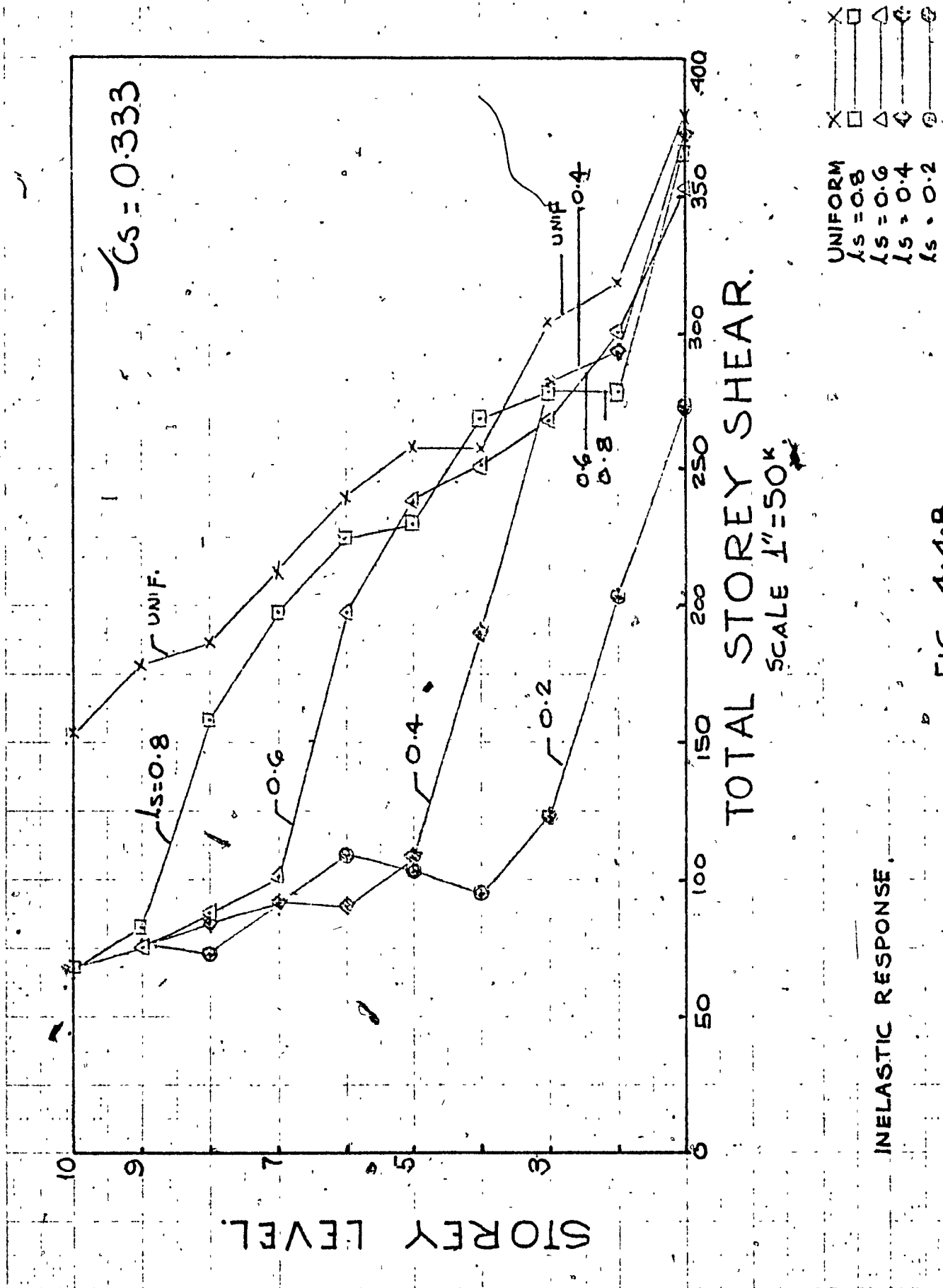
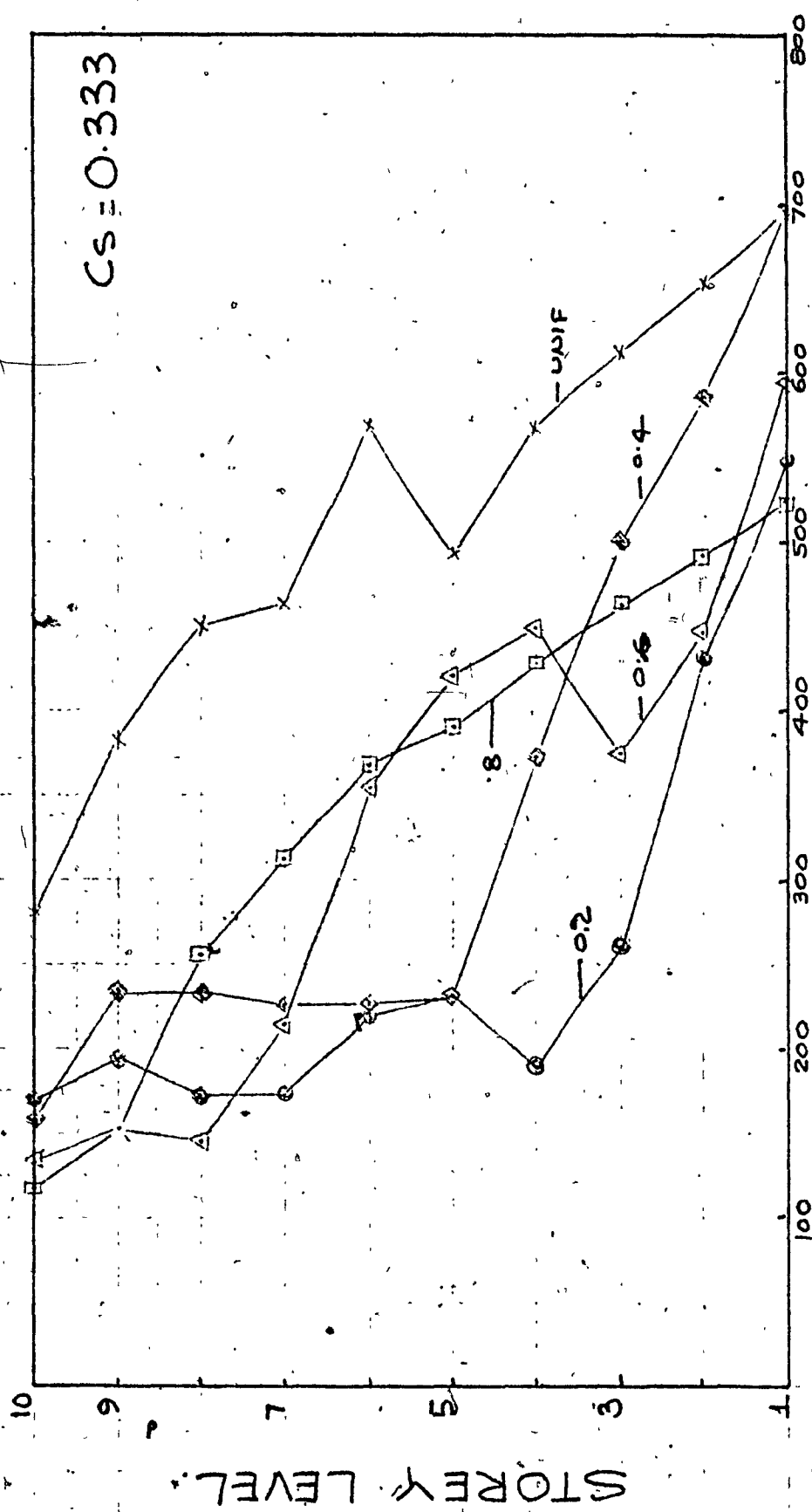


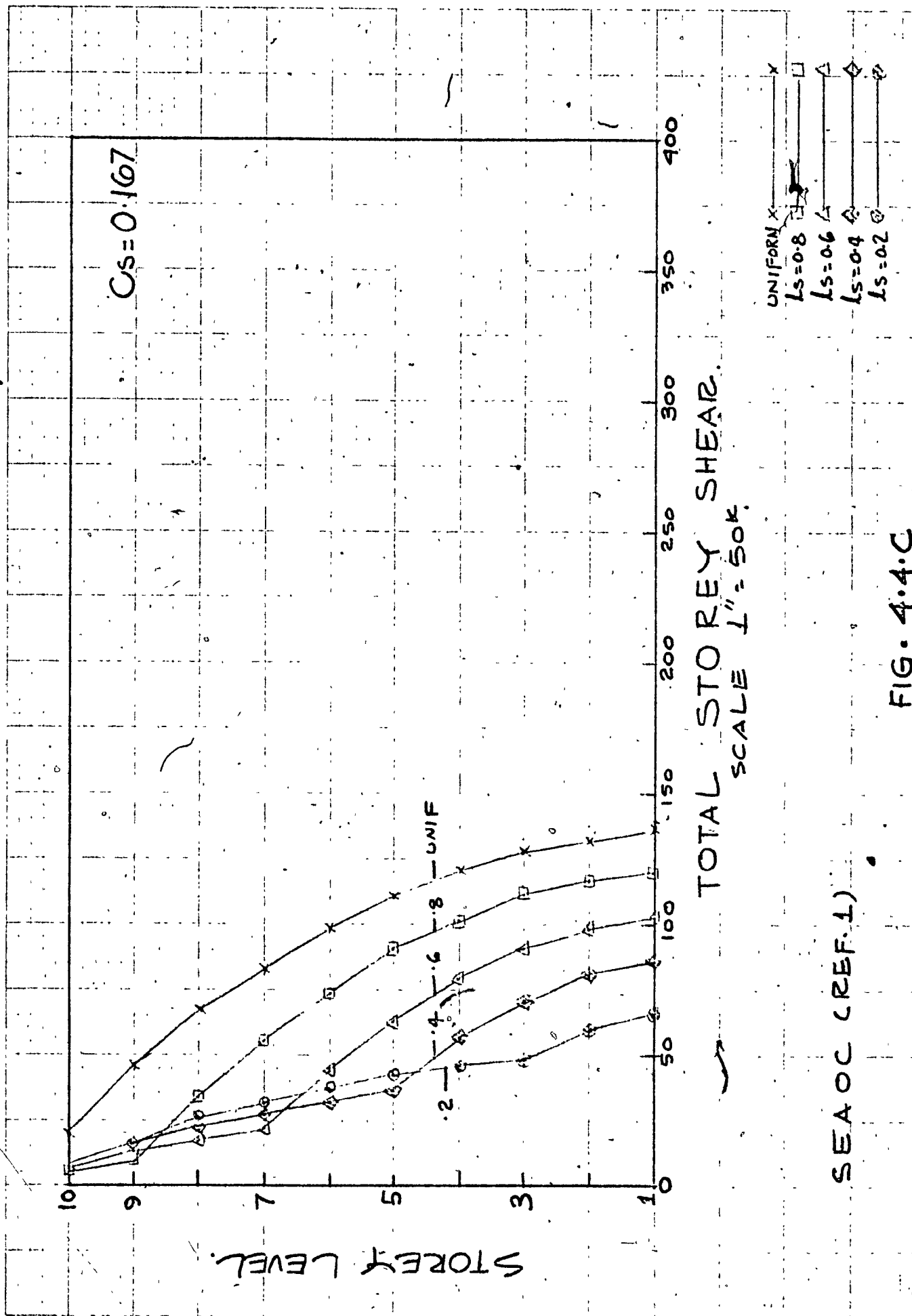
FIG. 4.4.B



TOTAL STOREY SHEAR.  
SCALE  $I'' = 100K$ .

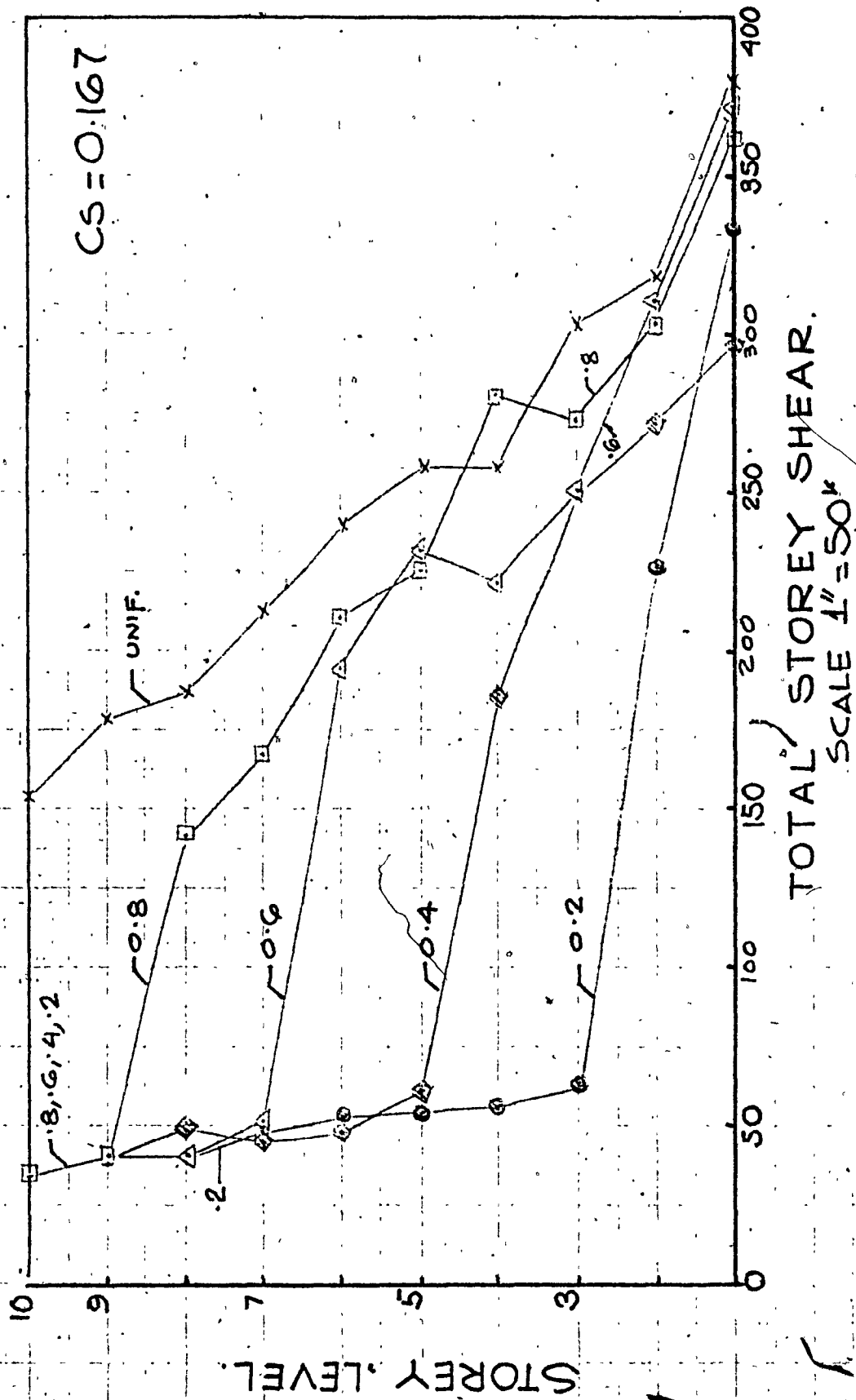
ELASTIC RESPONSE.

FIG. 4.4.B



SEAOC (REF. 1)

FIG. 4.4.C



UNIFORM

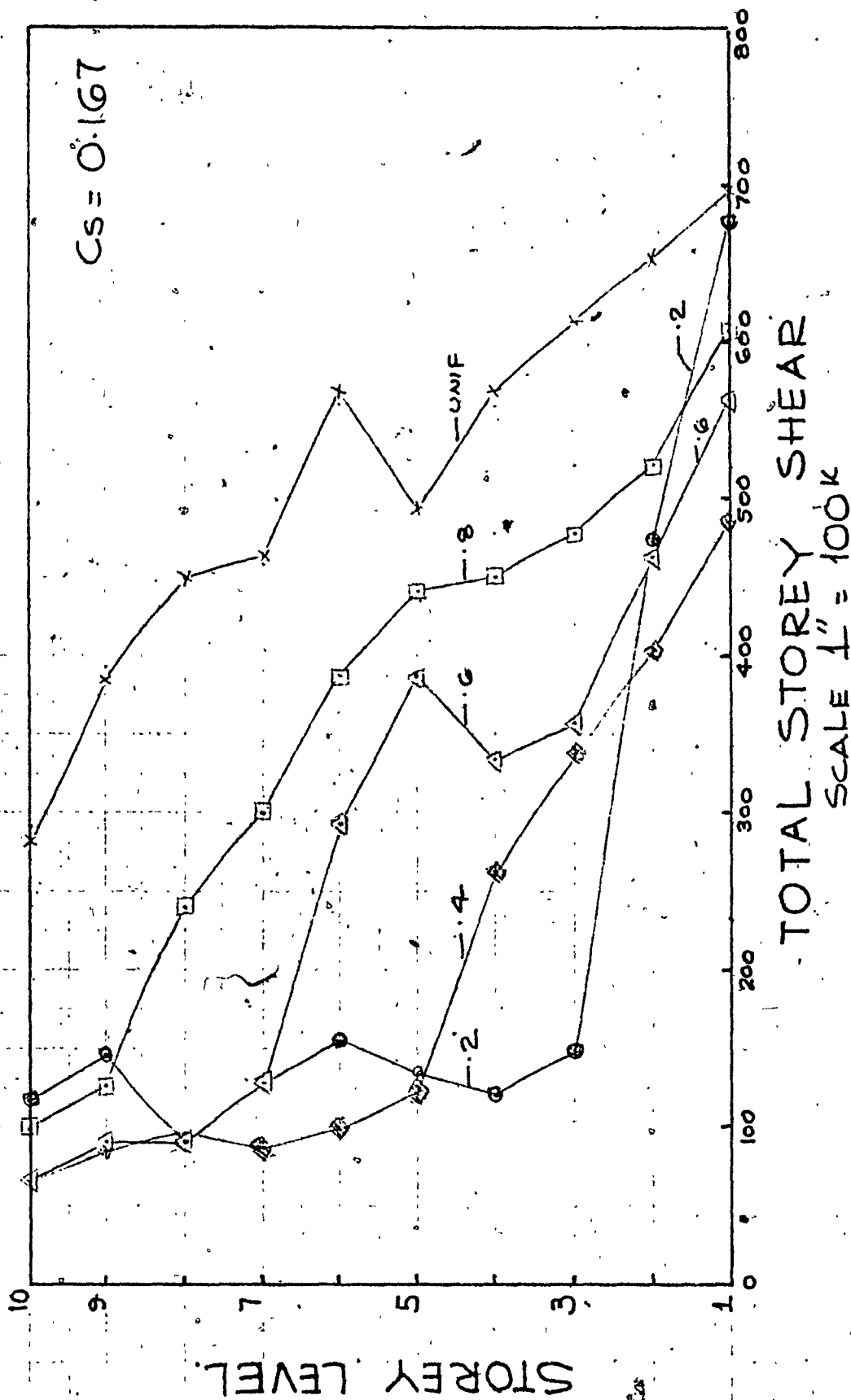
15 = 0.8

16 = 0.6

18 = 0.4

19 = 0.2

FIG. 4.4.C



UNIFORM (x)

1s:0.8 (□)

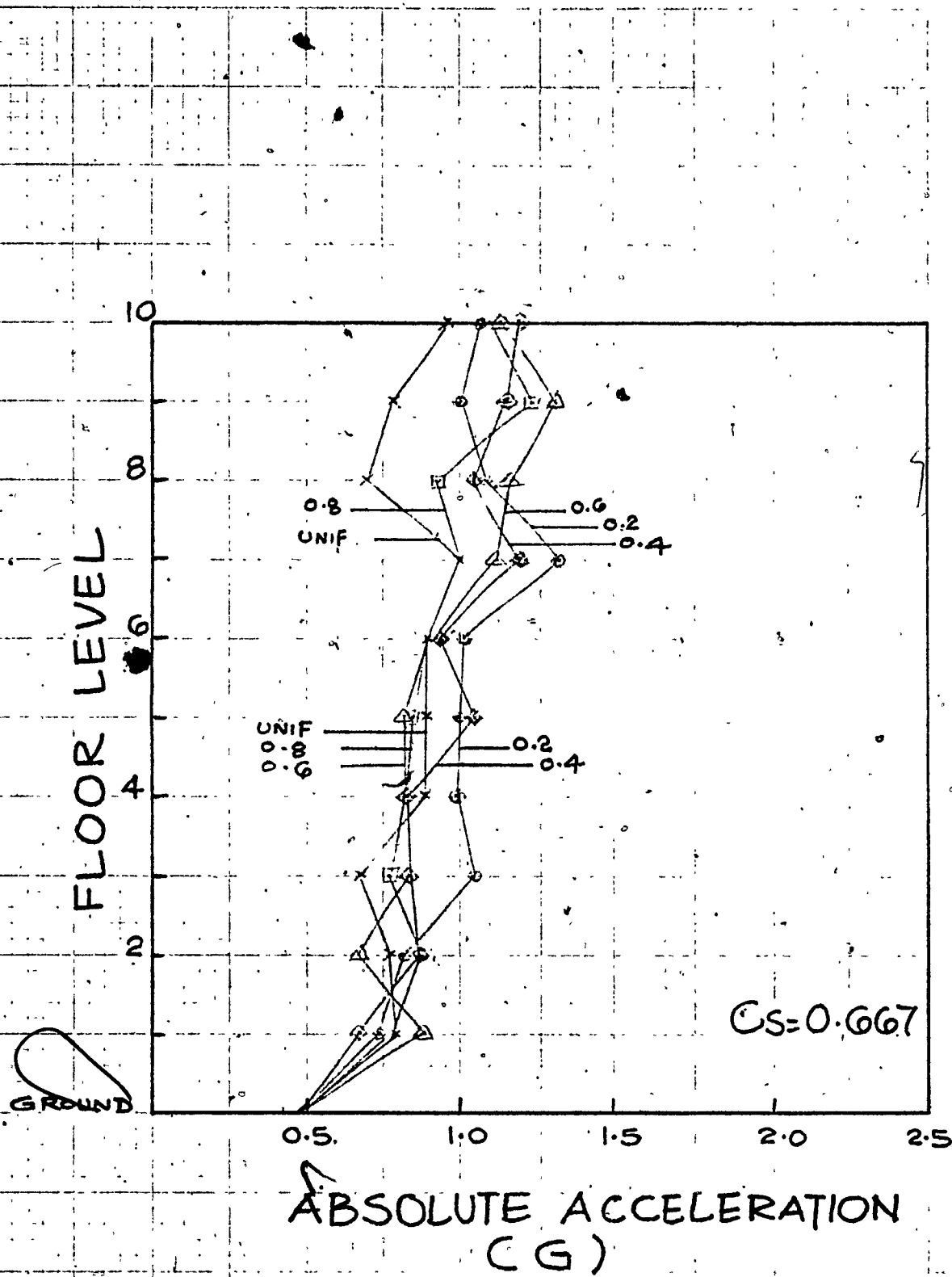
1s:0.6 (△)

1s:0.4 (◇)

1s:0.2 (○)

ELASTIC RESPONSE.

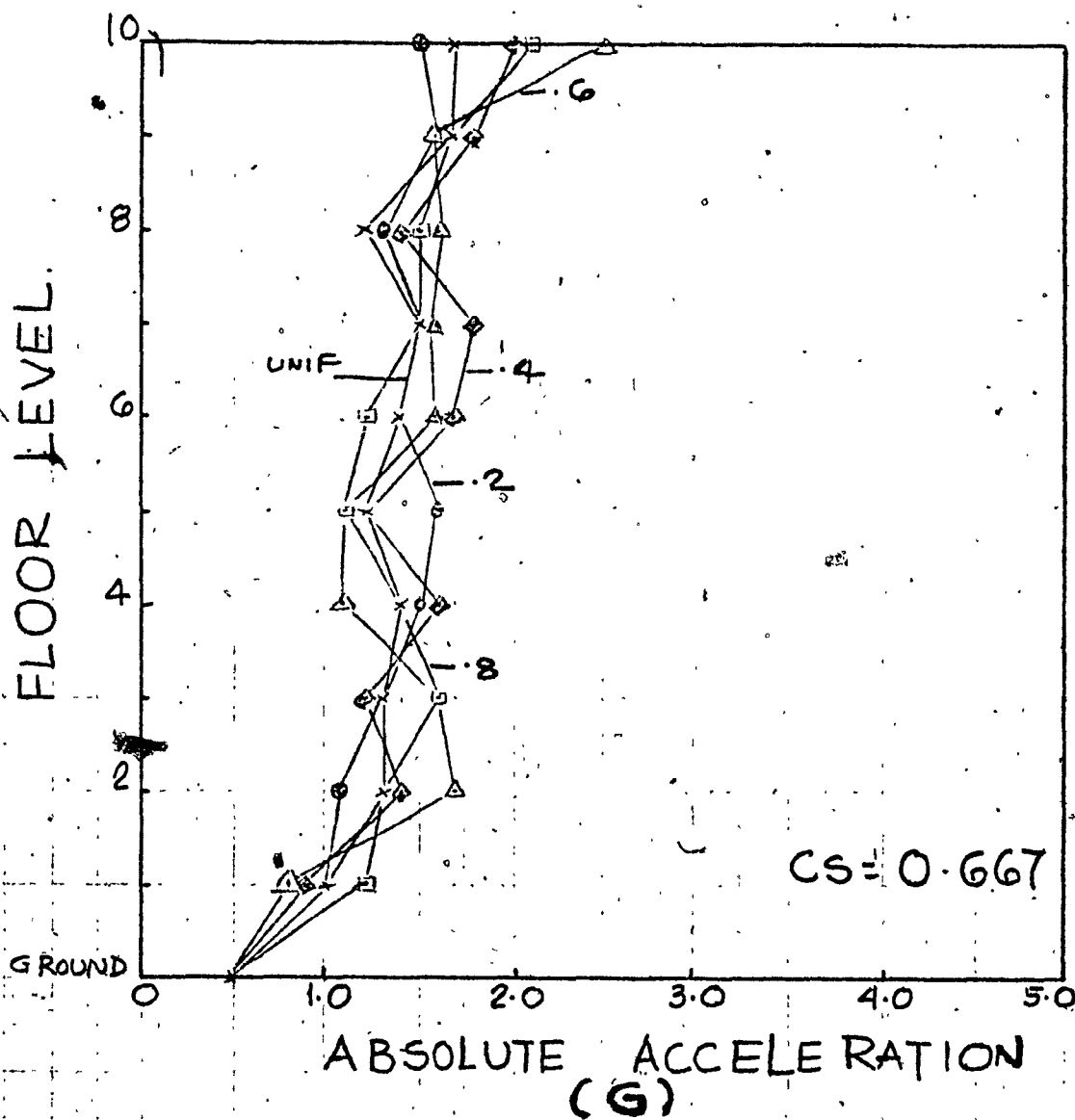
FIG. 4.4.C



INELASTIC RESPONSE

UNIFORM  
 $I_s = 0.8$    
 $I_s = 0.6$    
 $I_s = 0.4$    
 $I_s = 0.2$

FIG. 4.5.A

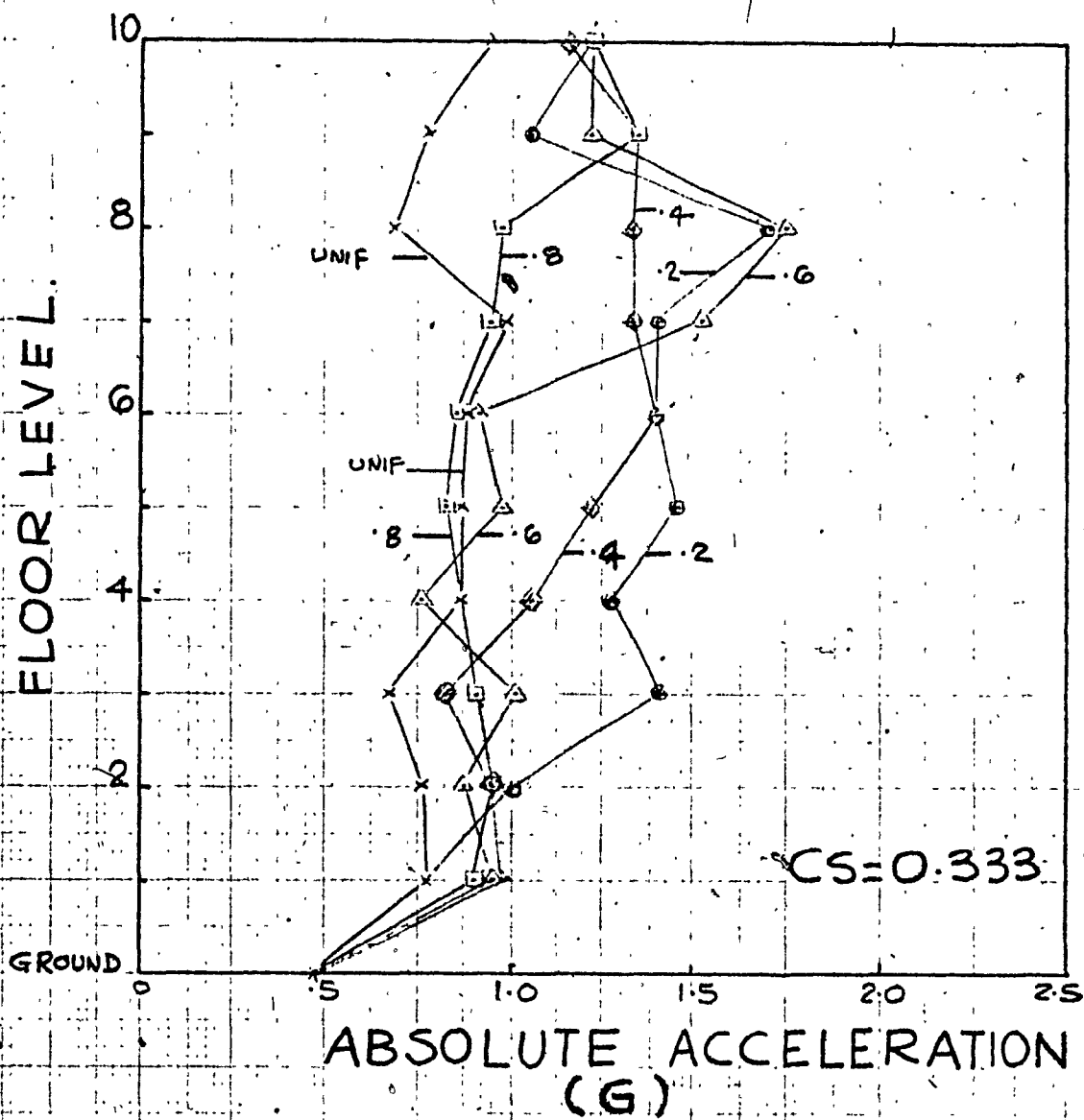


ELASTIC RESPONSE.

UNIFORM	X-----X
ls = 0.8	□-----□
ls = 0.6	△-----△
ls = 0.4	◇-----◇
ls = 0.2	○-----○

FIG. 4.5.A



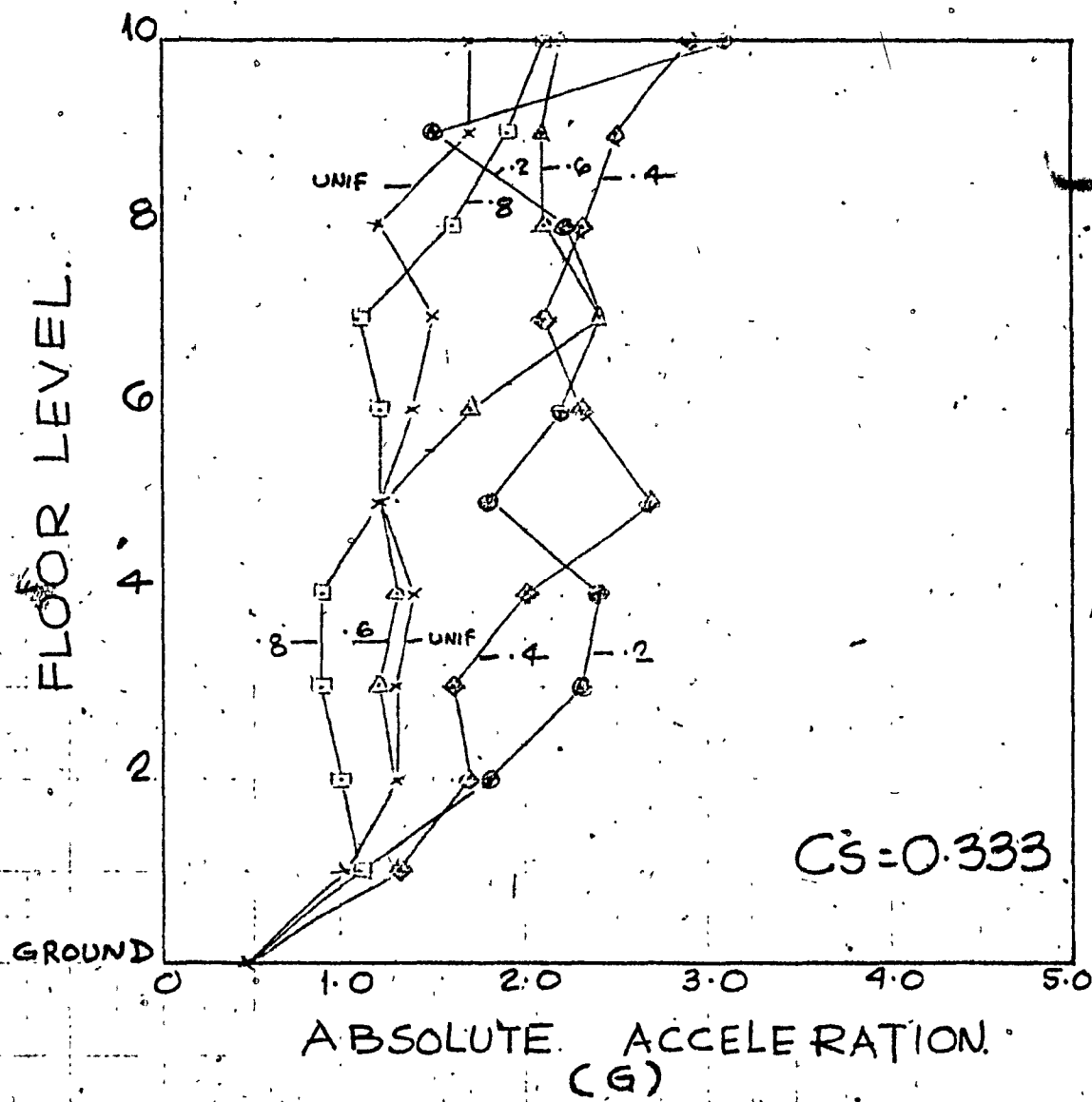


INELASTIC RESPONSE

FIG. 4.5.B

UNIFORM  
 $\zeta = 0.8$   
 $\zeta = 0.6$   
 $\zeta = 0.4$   
 $\zeta = 0.2$

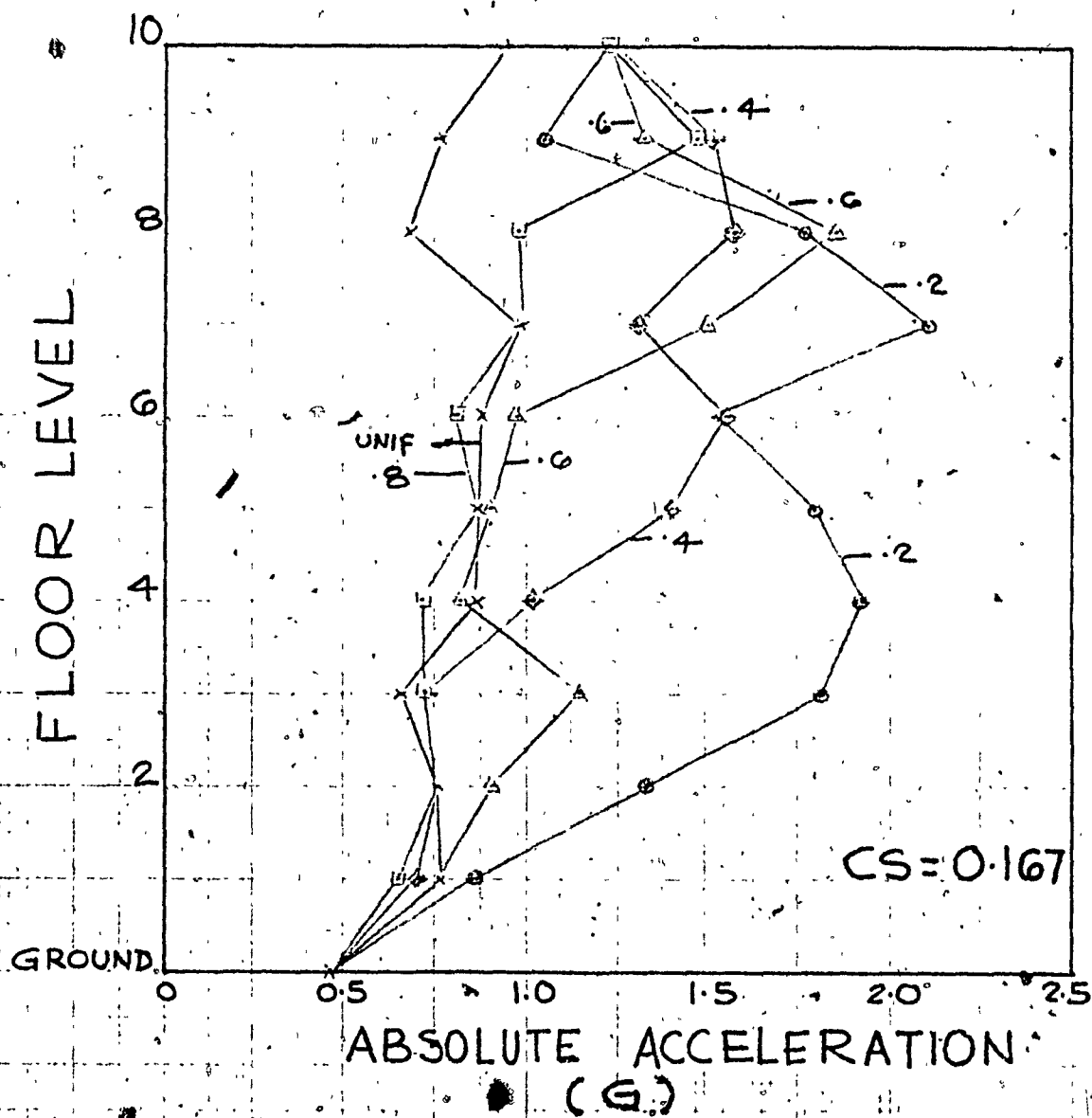
—x—x—  
 —□—□—  
 —△—△—  
 —◇—◇—  
 —⊕—⊕—



ELASTIC RESPONSE.

FIG. 4-5.B

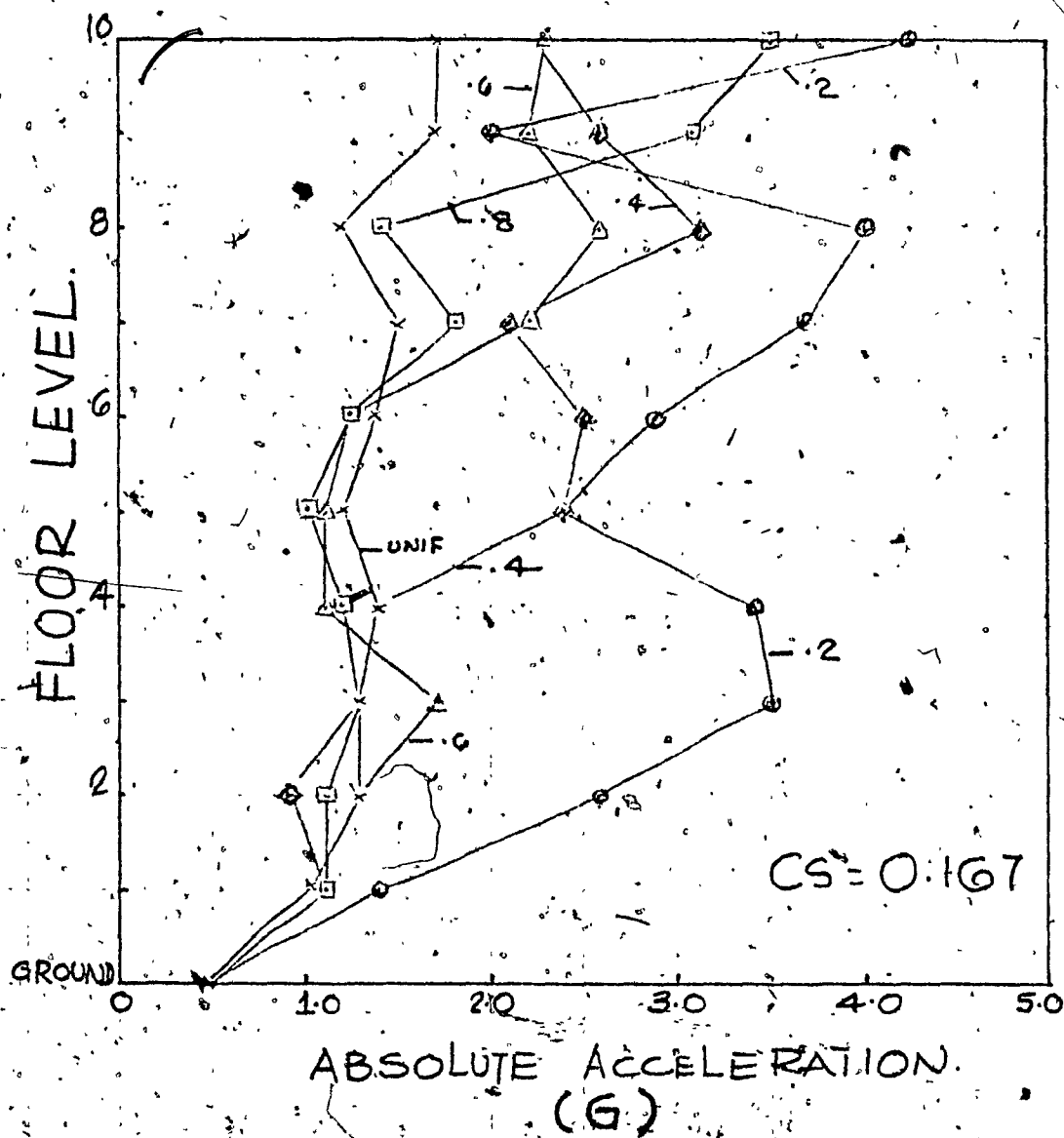
UNIFORM  $\times$ — $\times$   
 $l_s = 0.8$   $\square$ — $\square$   
 $l_s = 0.6$   $\triangle$ — $\triangle$   
 $l_s = 0.4$   $\diamond$ — $\diamond$   
 $l_s = 0.2$   $\circ$ — $\circ$



INELASTIC RESPONSE

UNIFORM  $\times$  —  $\times$   
 $l_s = 0.8$   $\square$  —  $\square$   
 $l_s = 0.6$   $\triangle$  —  $\triangle$   
 $l_s = 0.4$   $\diamond$  —  $\diamond$   
 $l_s = 0.2$   $\circ$  —  $\circ$

FIG. 4.5-C



ELASTIC RESPONSE

FIG. 4.5.C

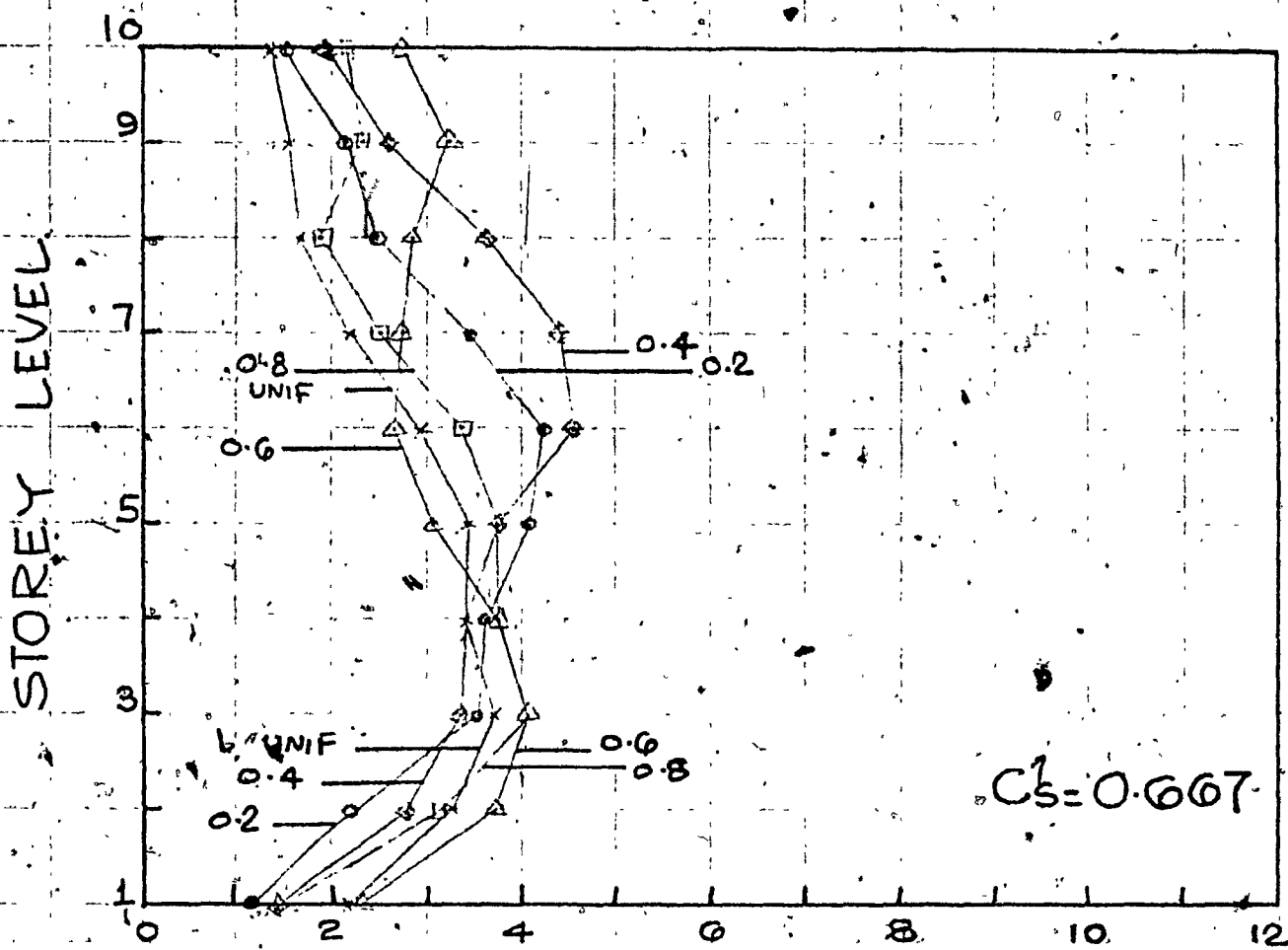
UNIFORM X — X

$l_s = 0.8$  □ — □

$l_s = 0.6$  △ — △

$l_s = 0.4$  ◇ — ◇

$l_s = 0.2$  ○ — ○

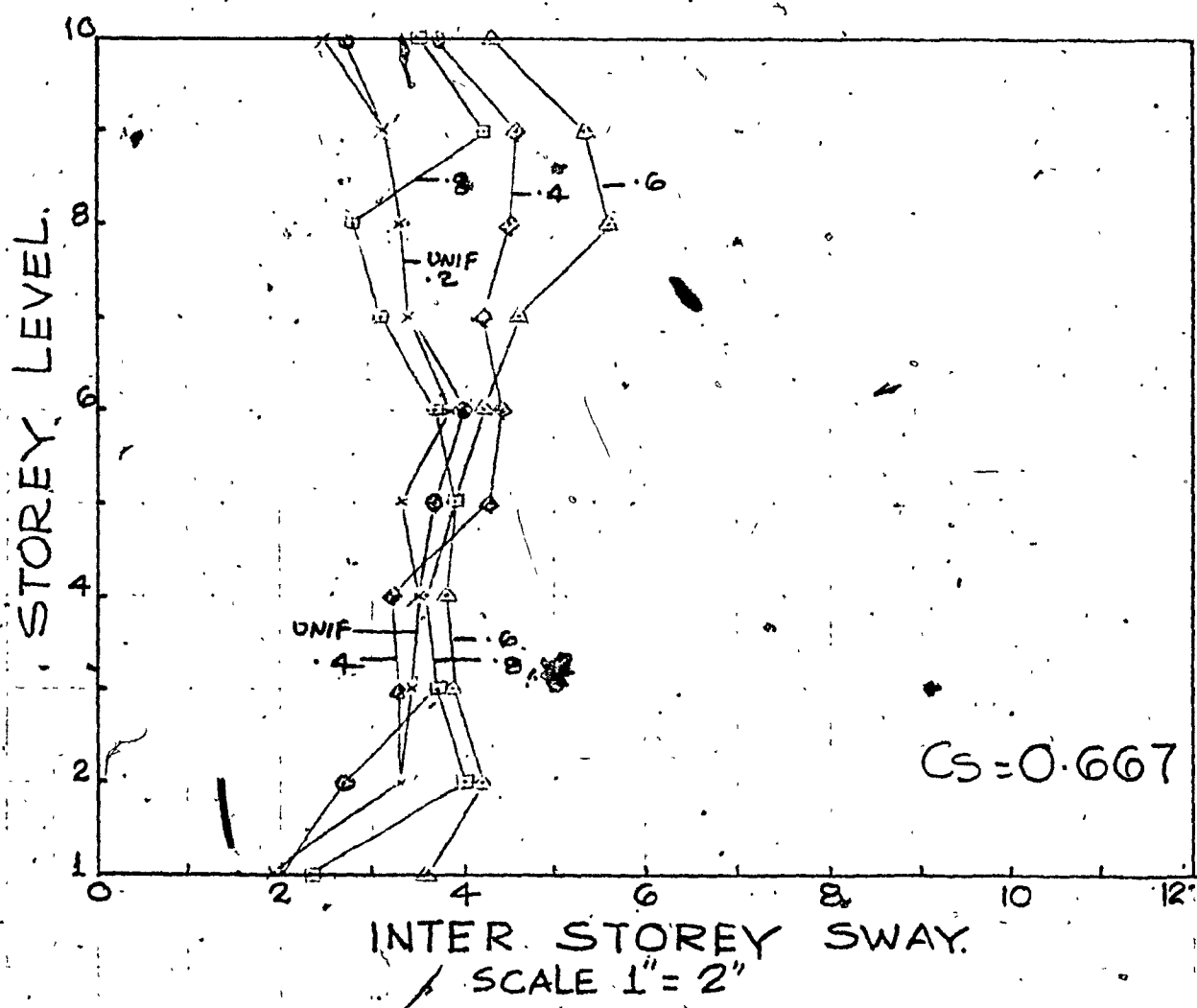


INTER STOREY SWAY  
SCALE 1" = 2"

INELASTIC RESPONSE

FIG. 4.6A

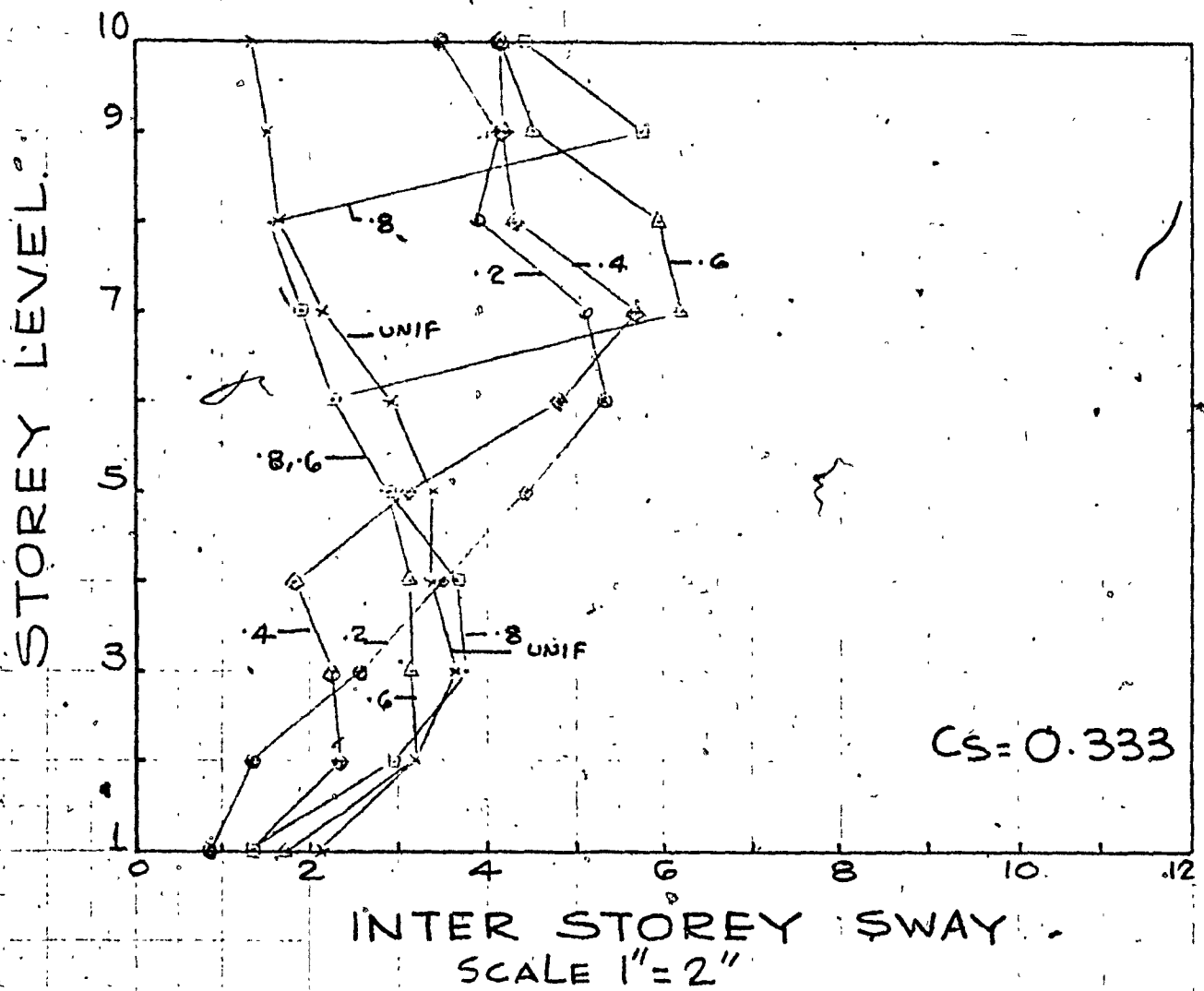
UNIFORM	X—X
$l_s = 0.8$	□—□
$l_s = 0.6$	△—△
$l_s = 0.4$	◇—◇
$l_s = 0.2$	○—○



ELASTIC RESPONSE.

FIG. 46.A

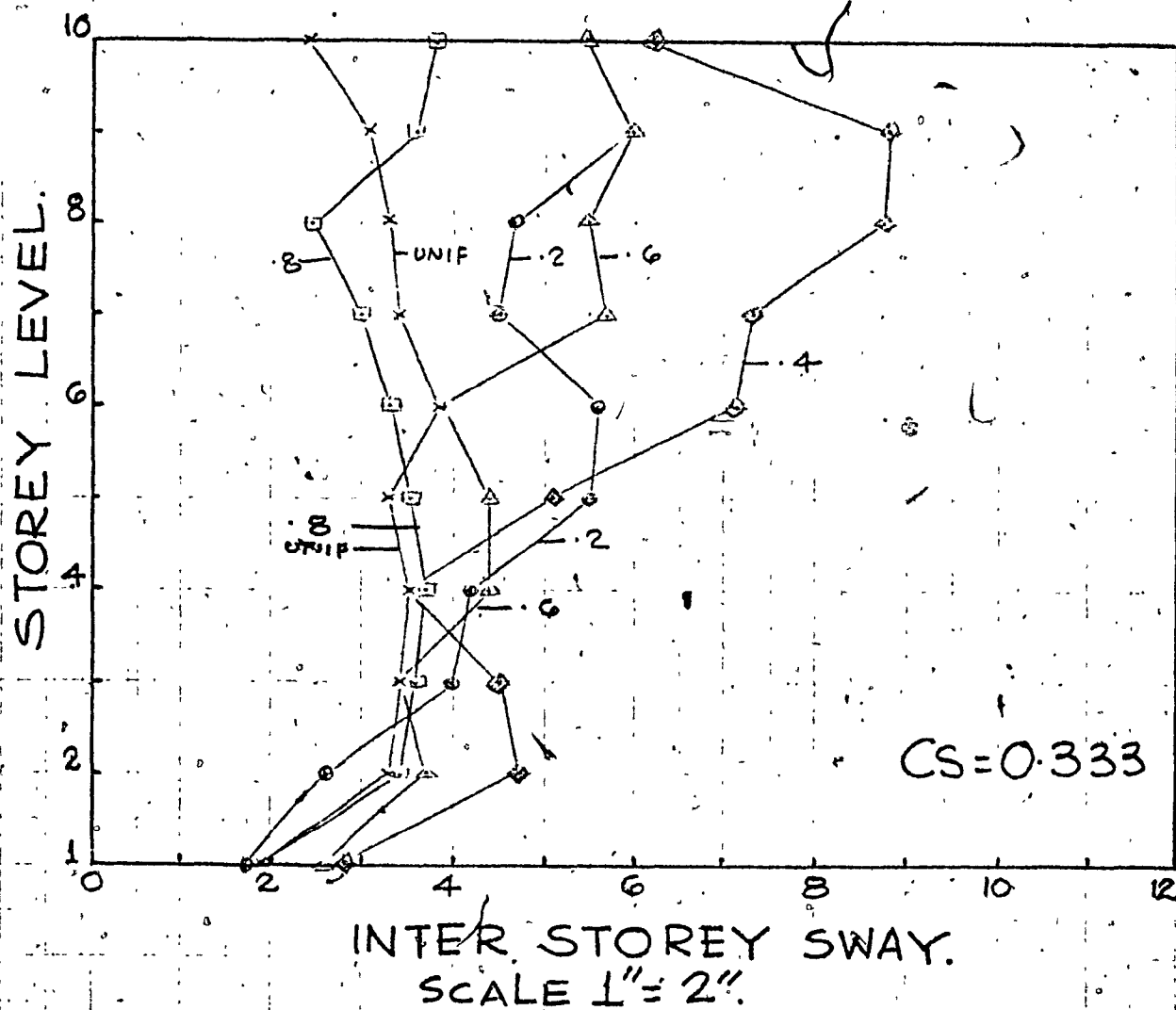
- UNIFORM x—x
- $l_s:0.8$  □—□
- $l_s:0.6$  △—△
- $l_s:0.4$  ◇—◇
- $l_s:0.2$  ○—○



INELASTIC RESPONSE

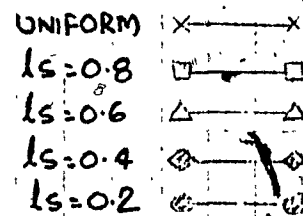
UNIFORM  $\times$ — $\times$   
 $I_s = 0.8$   $\square$ — $\square$   
 $I_s = 0.6$   $\triangle$ — $\triangle$   
 $I_s = 0.4$   $\diamond$ — $\diamond$   
 $I_s = 0.2$   $\bullet$ — $\bullet$

FIG. 4.6.B

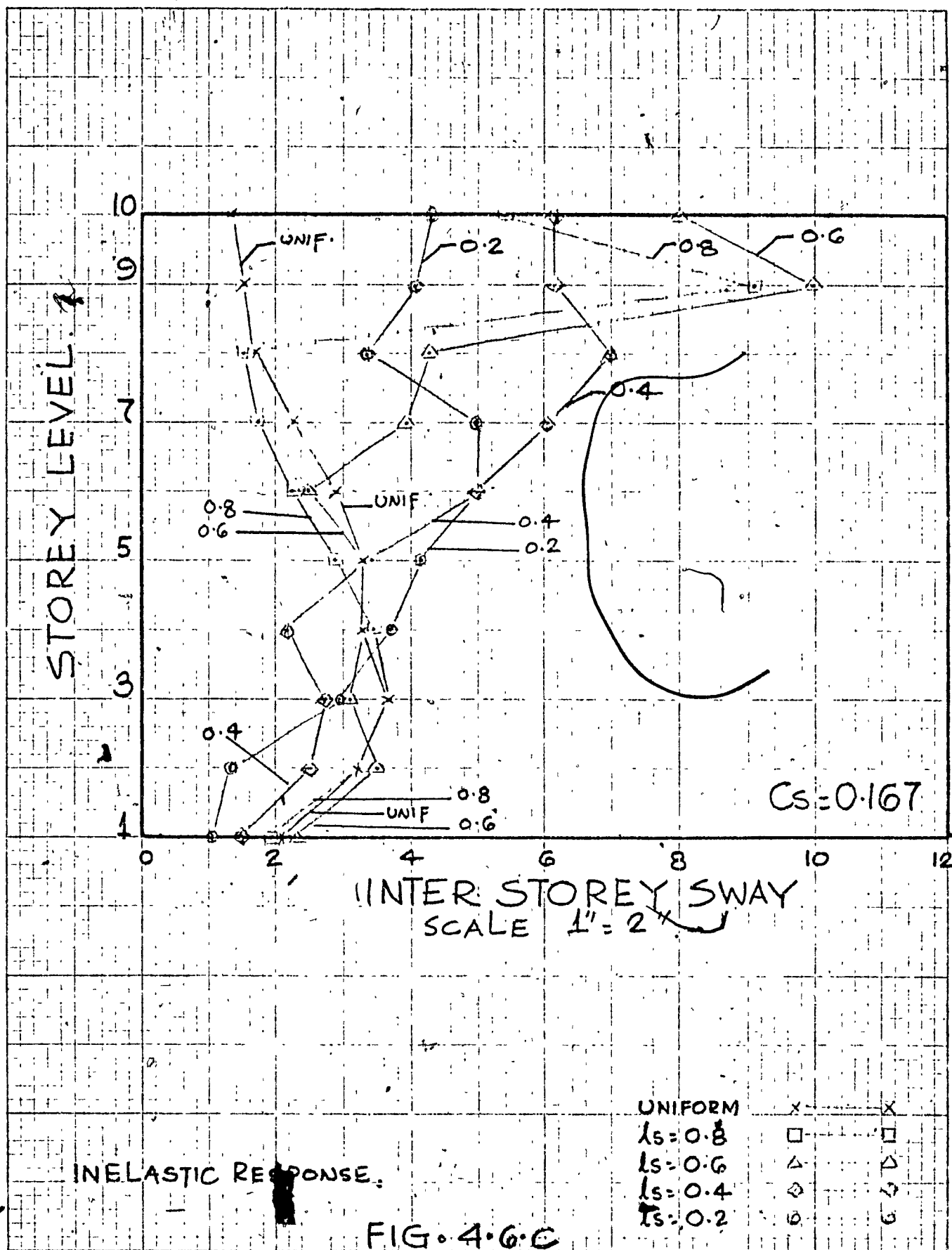


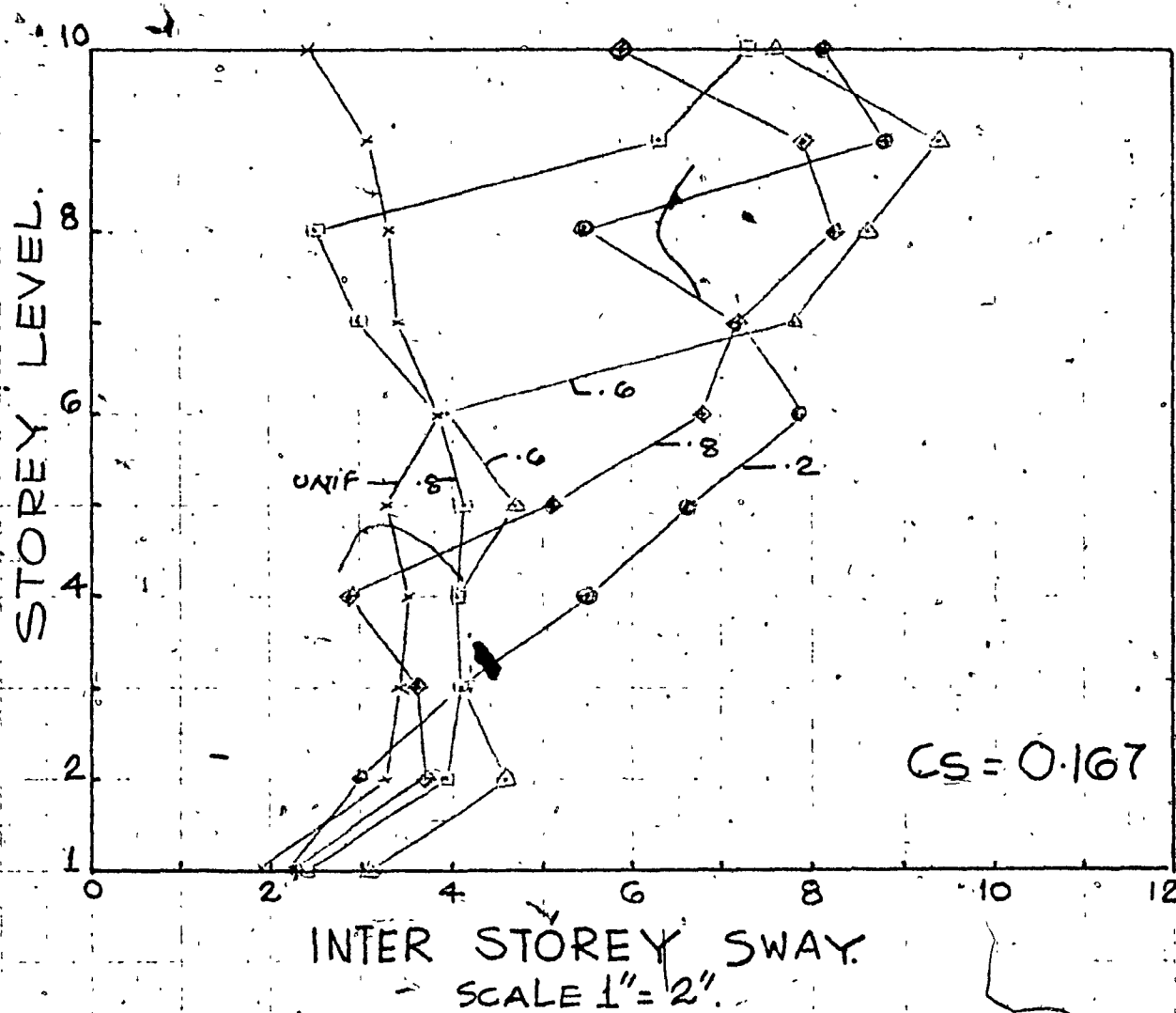
ELASTIC RESPONSE.

FIG. 4.6.B









ELASTIC RESPONSE.

UNIFORM X—X

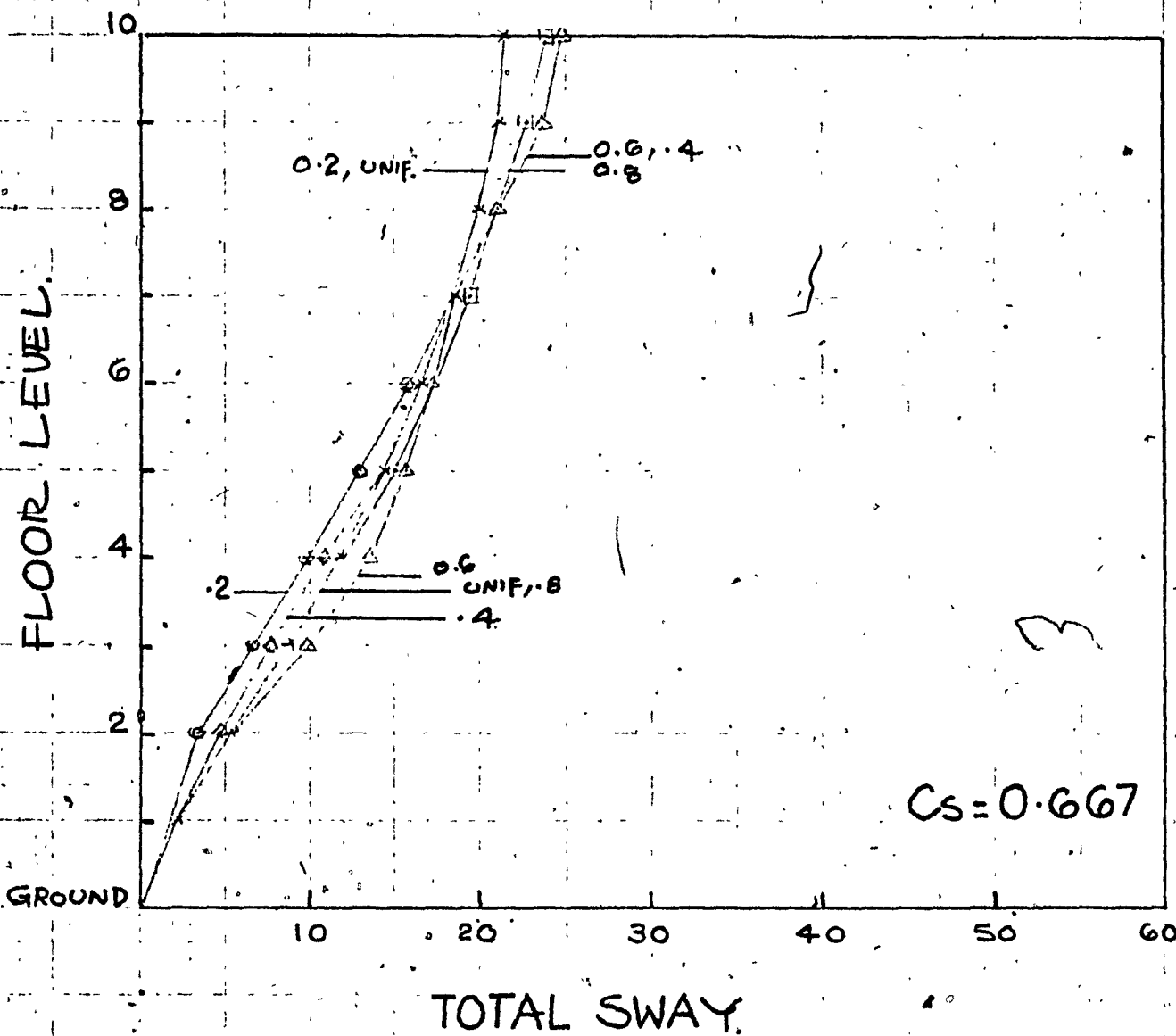
ls:0.8 □—□

ls:0.6 △—△

ls:0.4 ◇—◇

ls:0.2 ○—○

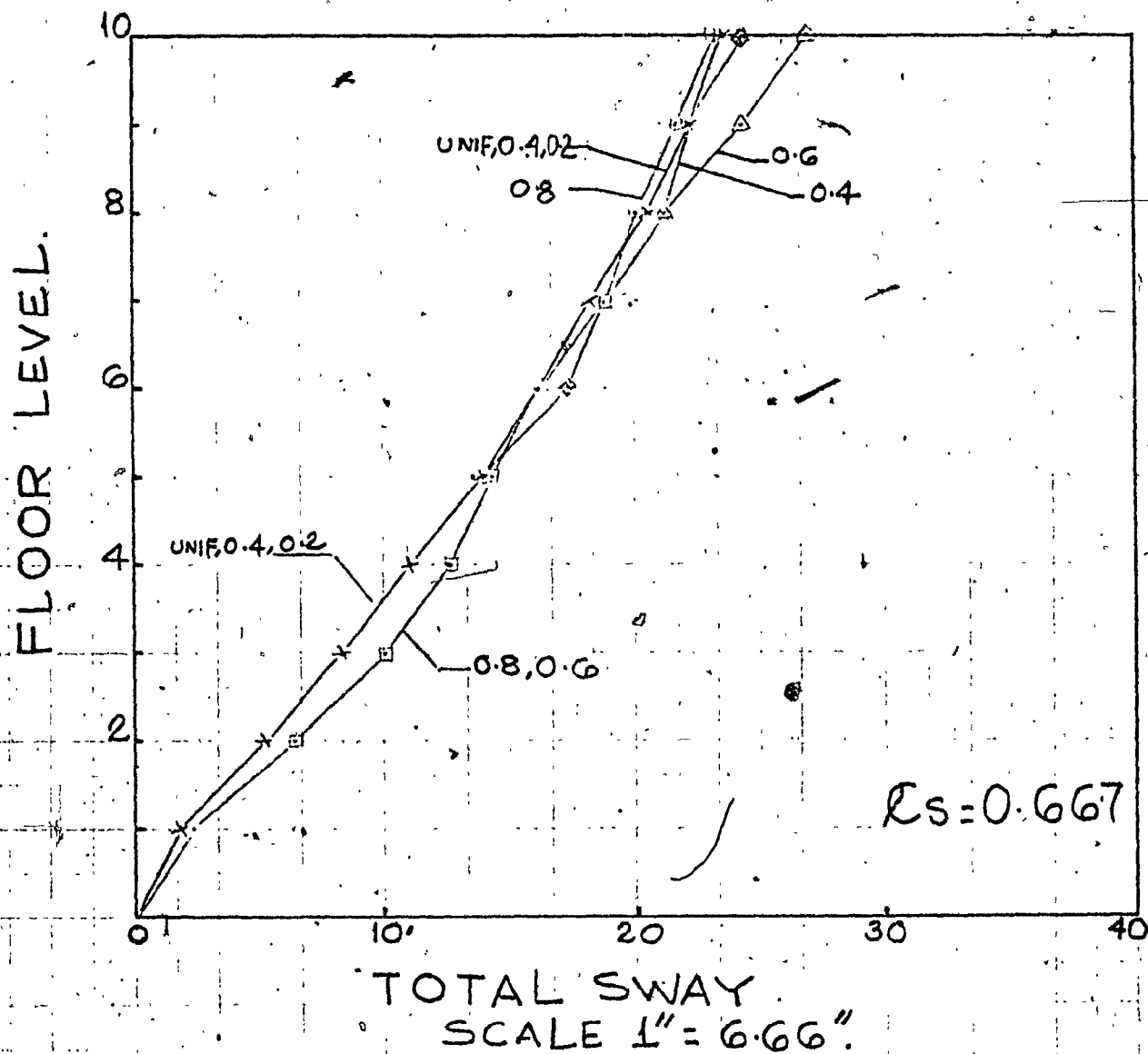
FIG. 4.6.C



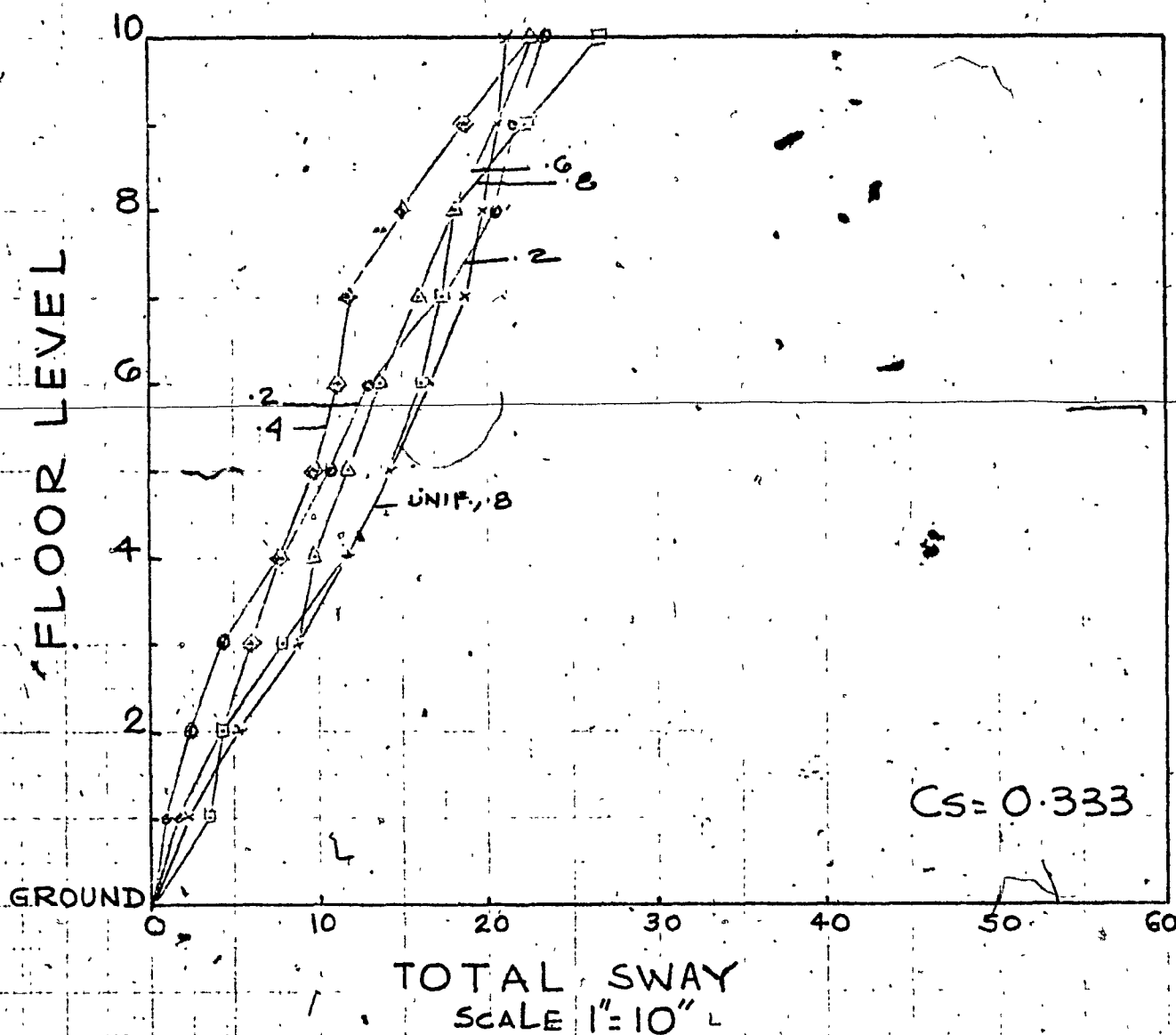
INELASTIC RESPONSE.

UNIFORM	
$l_s = 0.8$	$\square$
$l_s = 0.6$	$\triangle$
$l_s = 0.4$	$\diamond$
$l_s = 0.2$	$\circ$

FIG. 4.7.A

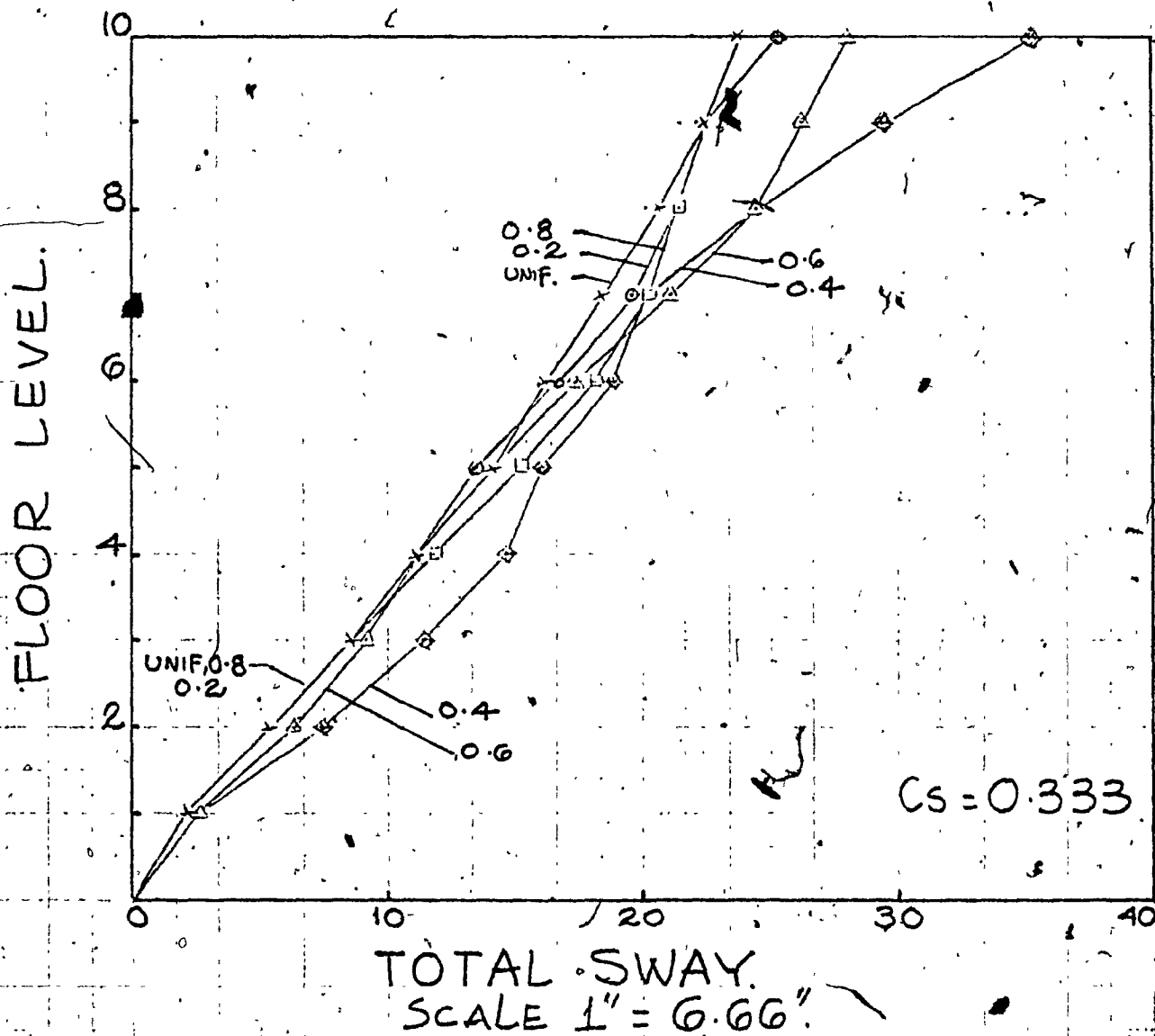


UNIFORM  $\times$  —  $\times$   
 $L_s = 0.8$   $\square$  —  $\square$   
 $L_s = 0.6$   $\triangle$  —  $\triangle$   
 $L_s = 0.4$   $\diamond$  —  $\diamond$   
 $L_s = 0.2$   $\oplus$  —  $\oplus$



INELASTIC RESPONSE.

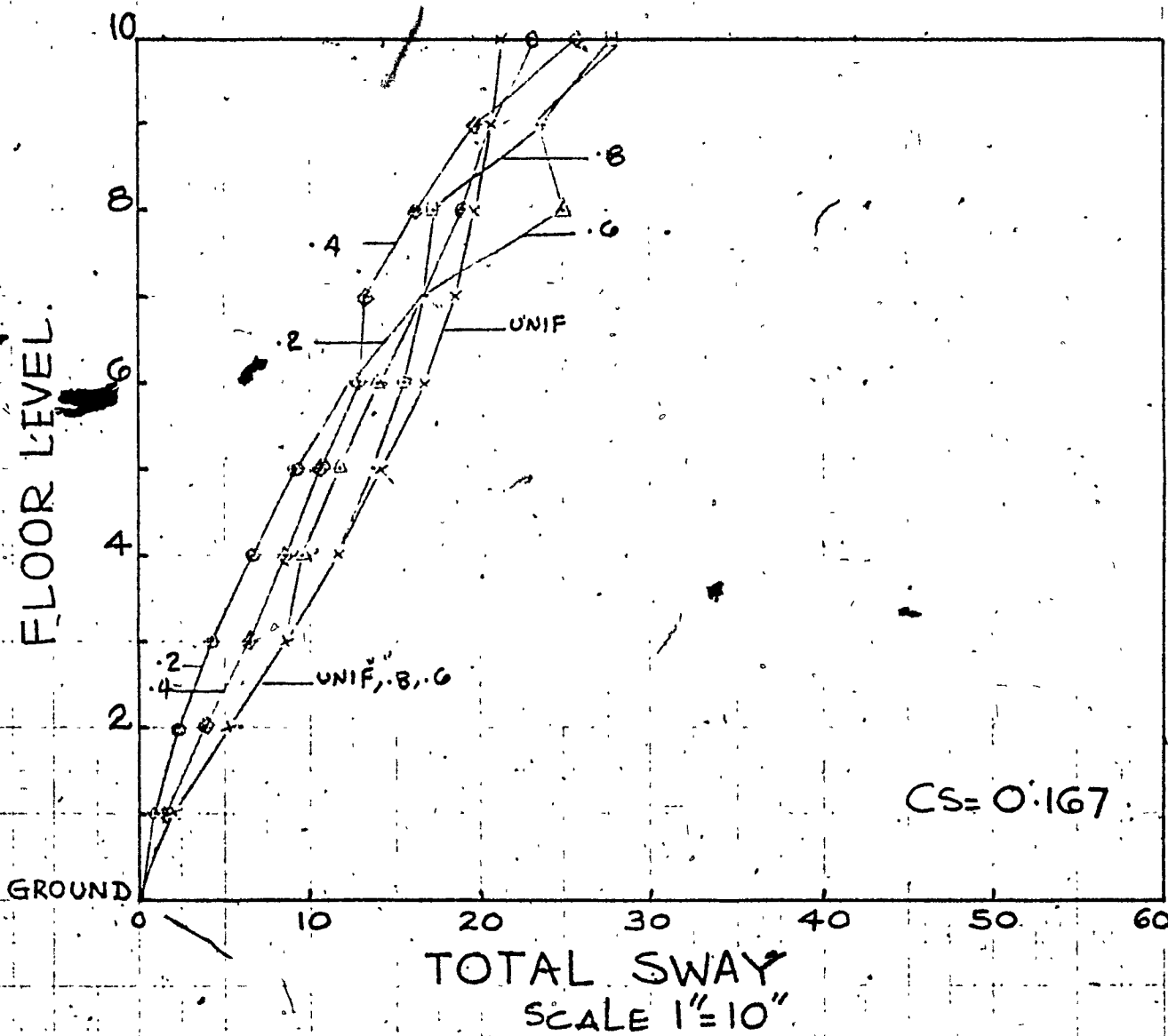
FIG. 4.7.B



ELASTIC RESPONSE

UNIFORM  $\times$ — $\times$   
 $l_s = 0.8$   $\square$ — $\square$   
 $l_s = 0.6$   $\triangle$ — $\triangle$   
 $l_s = 0.4$   $\diamond$ — $\diamond$   
 $l_s = 0.2$   $\circ$ — $\circ$

FIG. 4.7.B



INELASTIC RESPONSE

UNIFORM  
 $l_s = 0.8$   
 $l_s = 0.6$   
 $l_s = 0.4$   
 $l_s = 0.2$

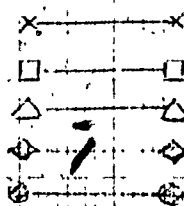
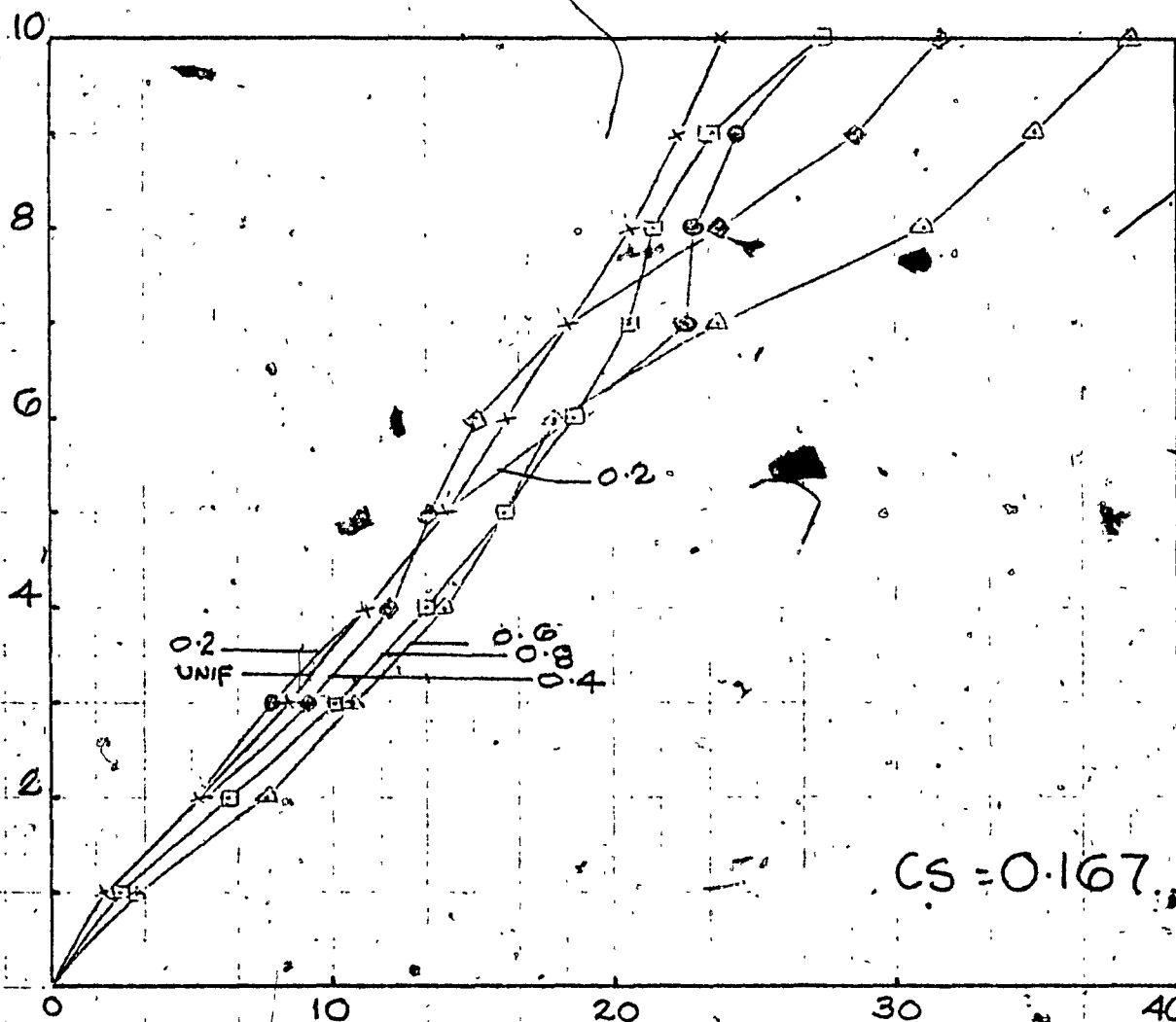


FIG. 4.7.C

FLOOR LEVEL



TOTAL SWAY.  
SCALE 1" = 6.66"

ELASTIC RESPONSE

FIG. 4.7-C

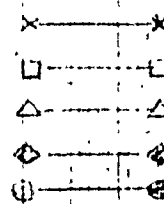
UNIFORM

ls:0.8

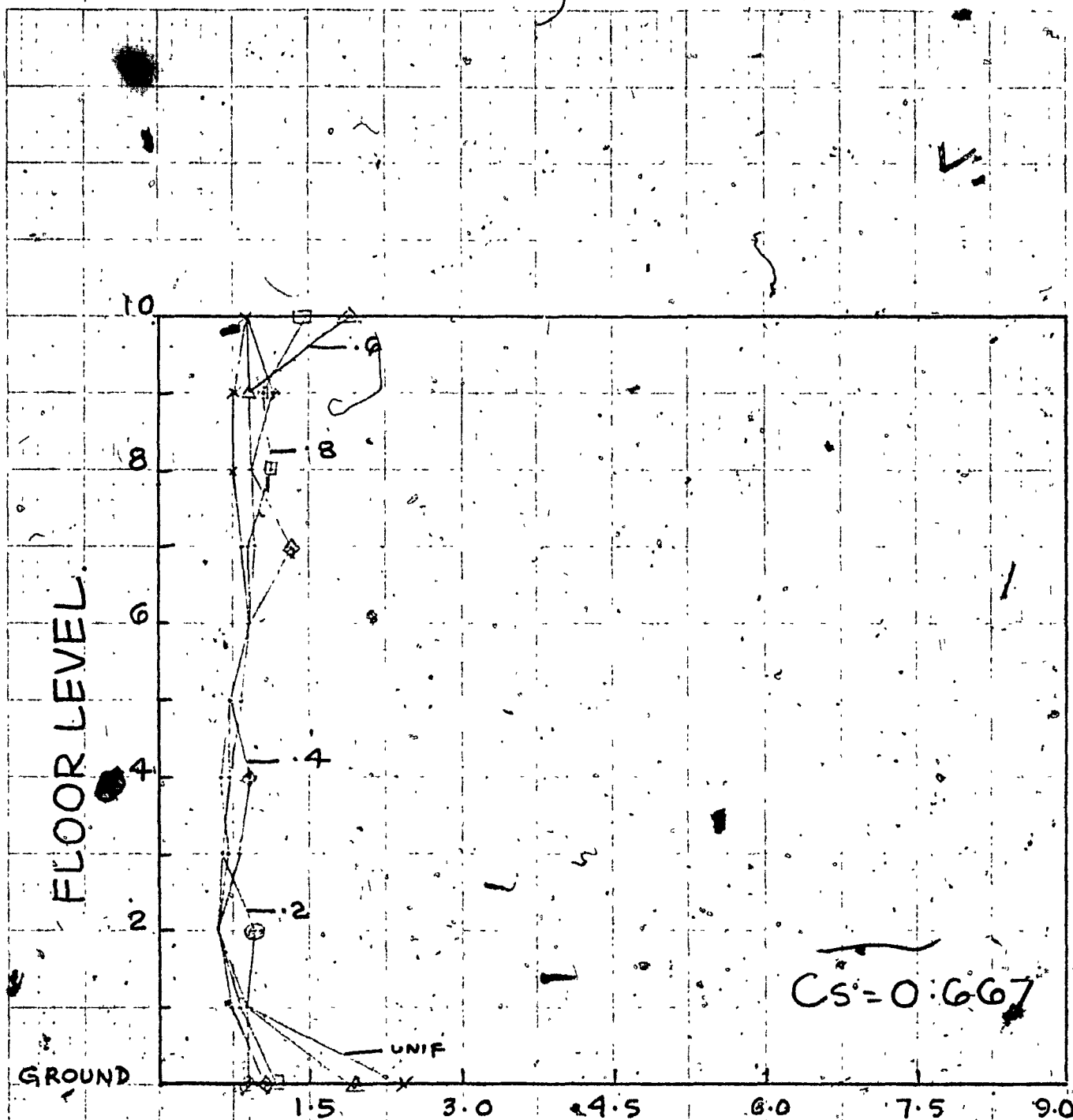
ls:0.6

ls:0.4

ls:0.2







COLUMN DUCTILITY FACTOR

SCALE 1"=1.5

INELASTIC RESPONSE

FIG. 4.8.A

UNIFORM

$L_s = 0.8$

$L_s = 0.6$

$L_s = 0.4$

$L_s = 0.2$

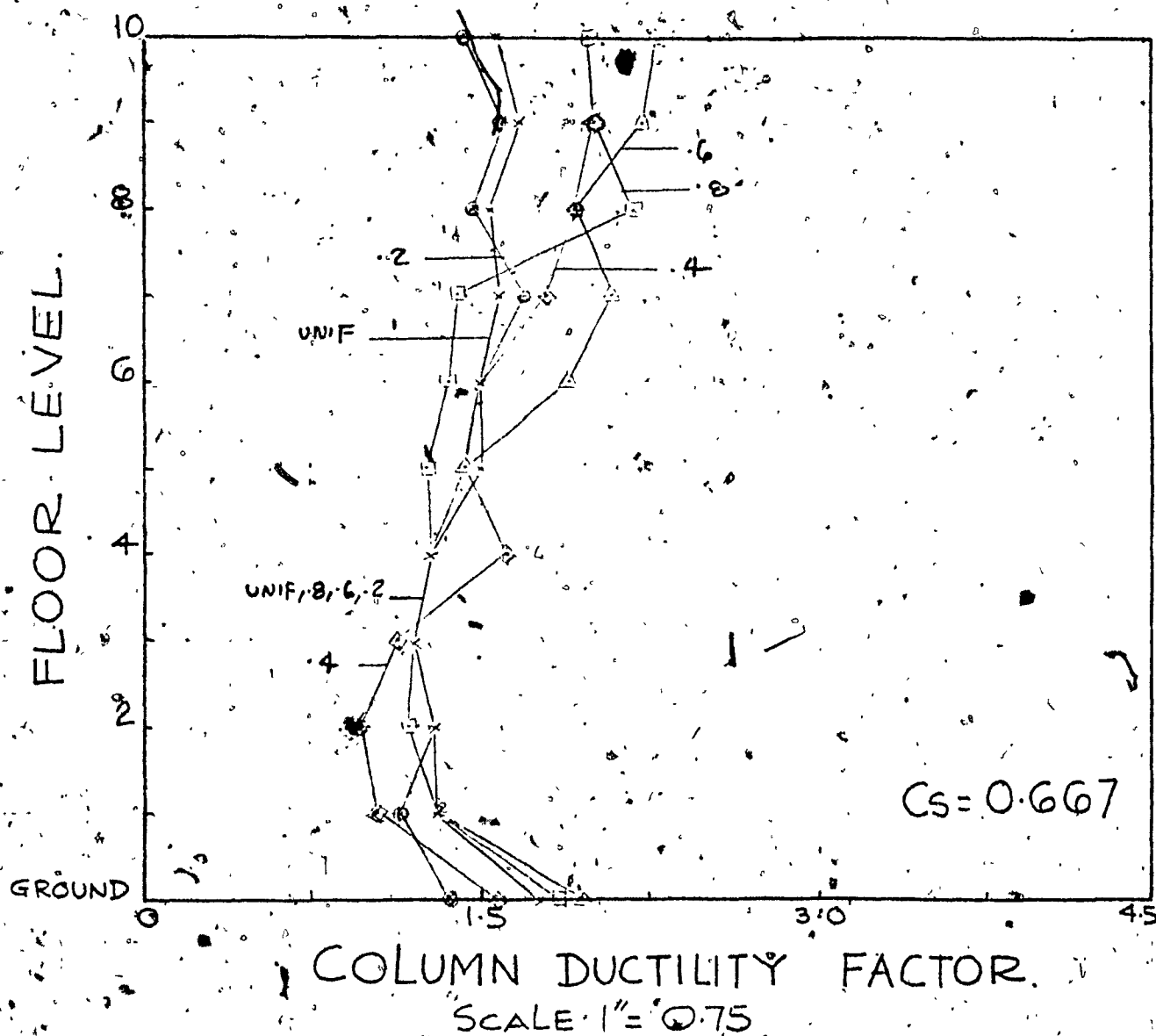
—x—x—

—□—□—

—△—△—

—◇—◇—

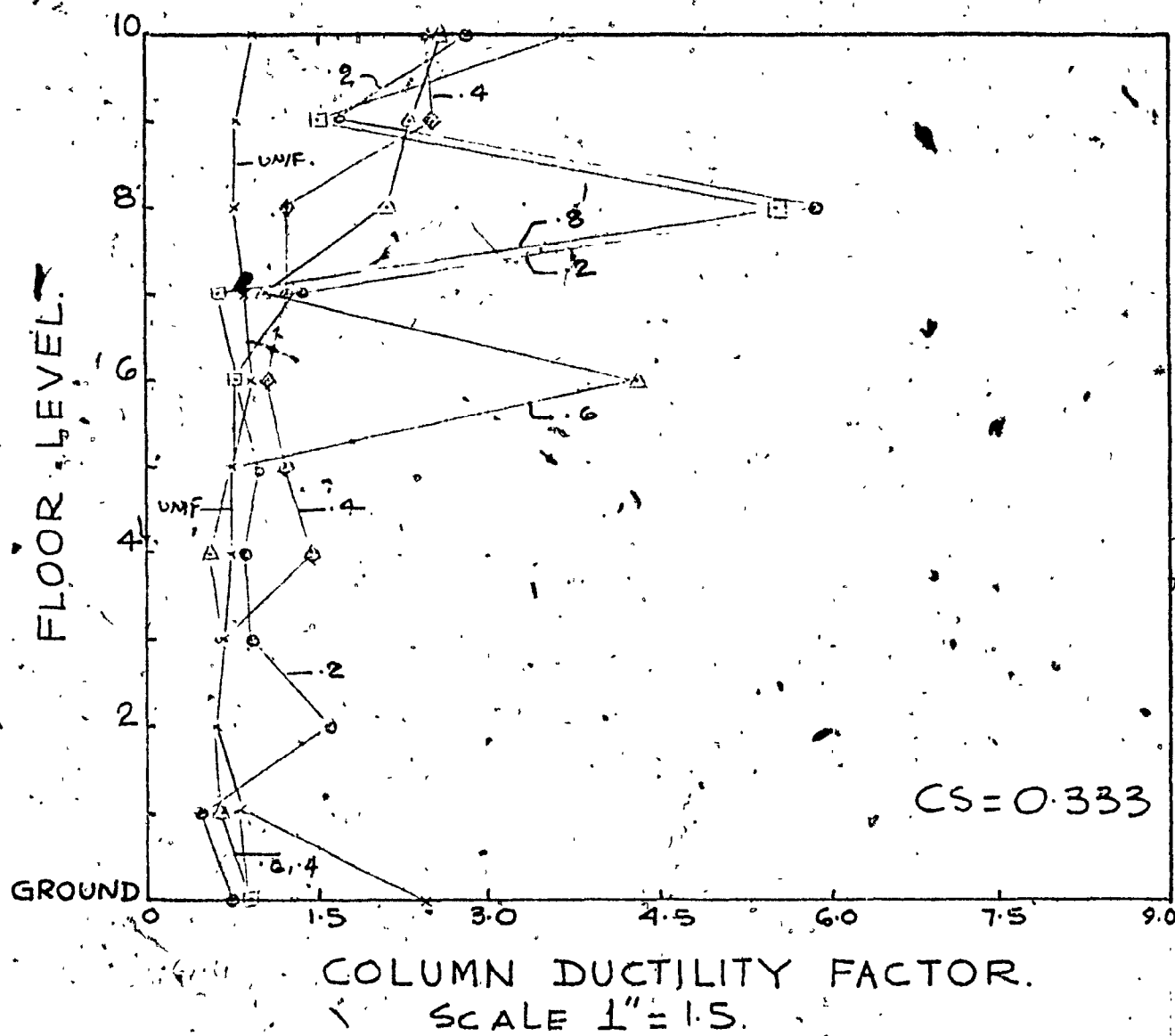
—○—○—



ELASTIC RESPONSE

UNIFORM ——— x  
 $I_s = 0.8$  □ ——— □  
 $I_s = 0.6$  △ ——— △  
 $I_s = 0.4$  ◇ ——— ◇  
 $I_s = 0.2$  ○ ——— ○

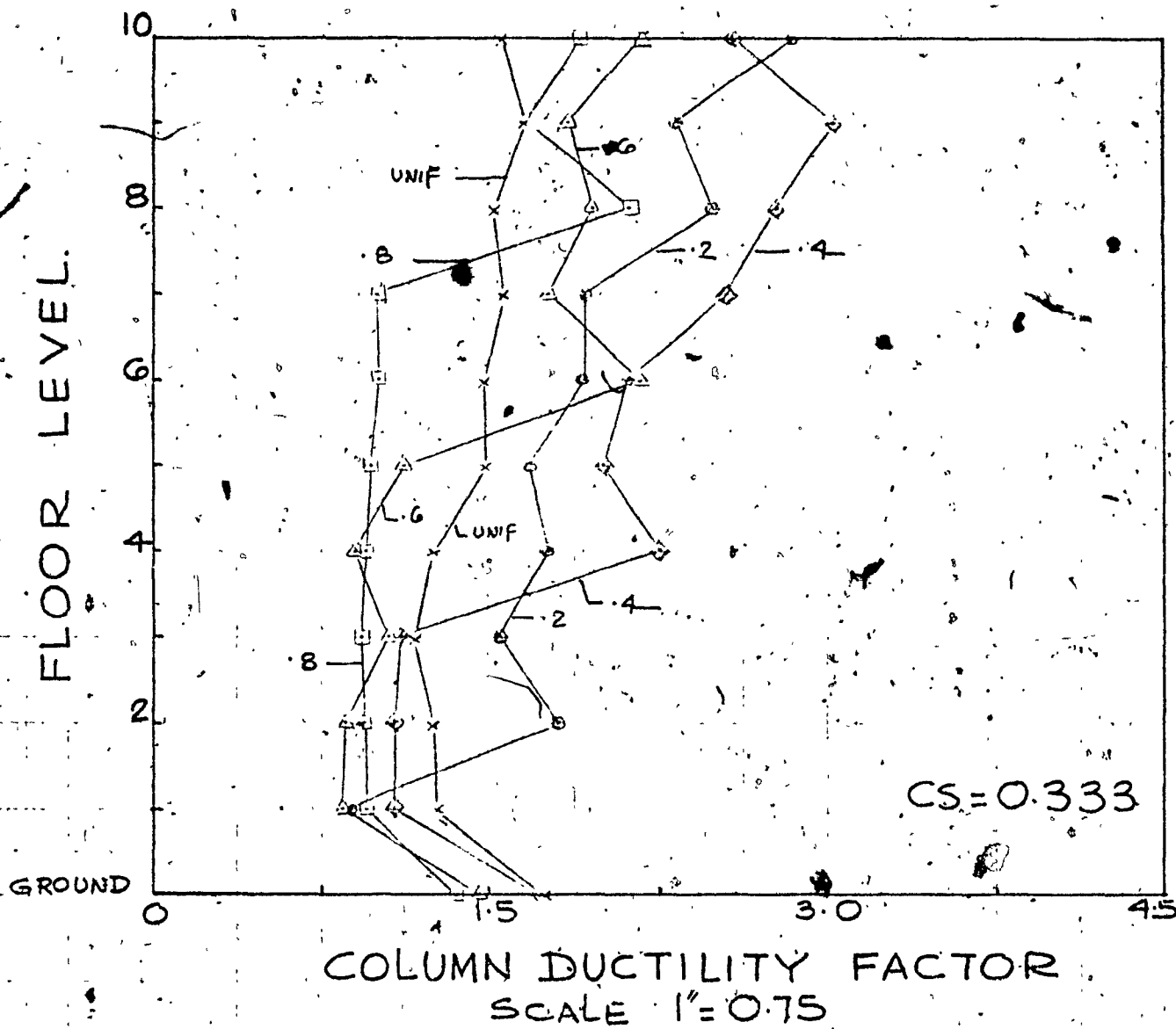
FIG. 4.8.A



INELASTIC RESPONSE

- |         |       |
|---------|-------|
| UNIFORM | x—x—x |
| ls=0.8  | □—□—□ |
| ls=0.6  | △—△—△ |
| ls=0.4  | ◇—◇—◇ |
| ls=0.2  | ○—○—○ |

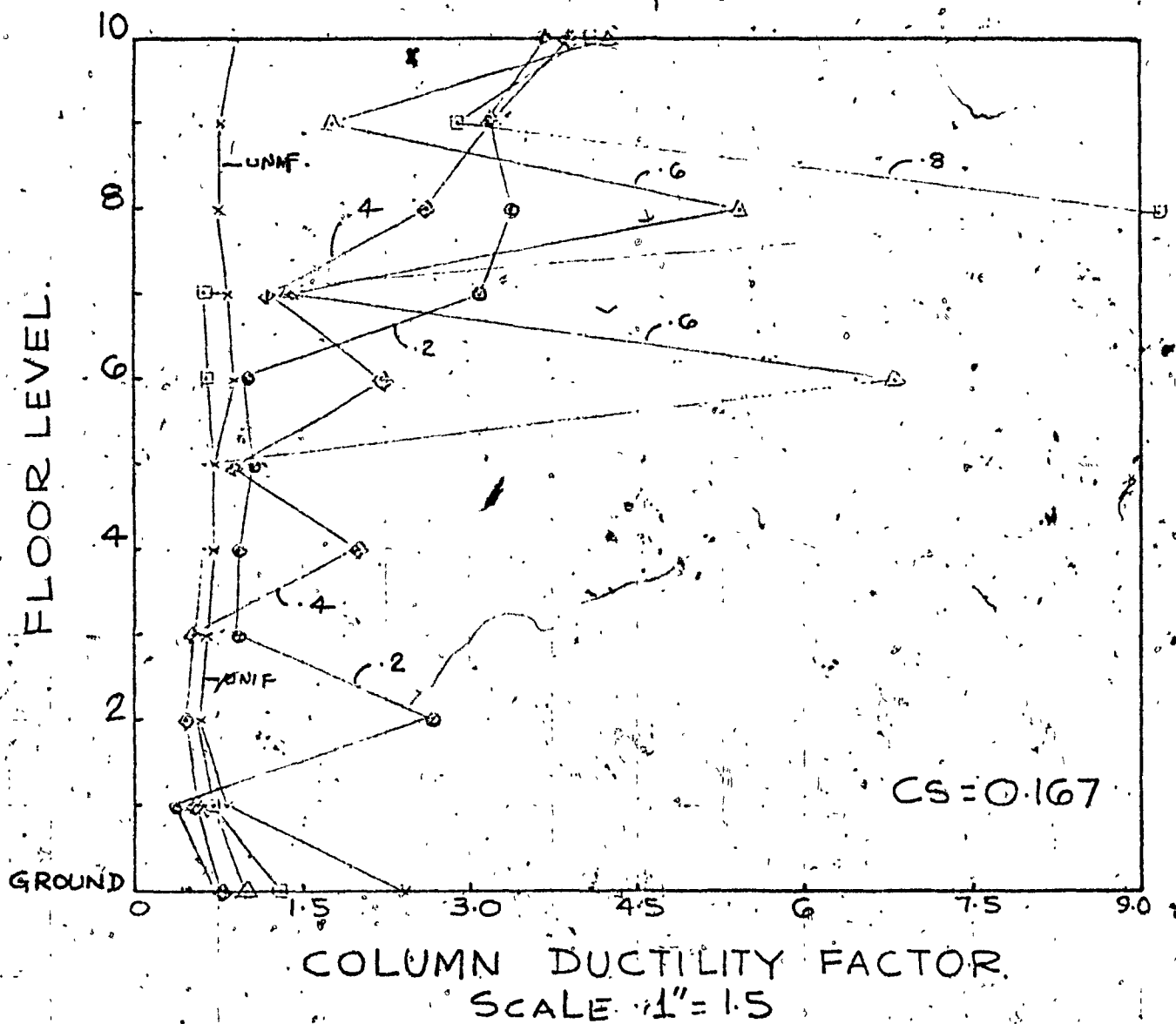
FIG. 4.8.B



ELASTIC RESPONSE

FIG. 4.8.B

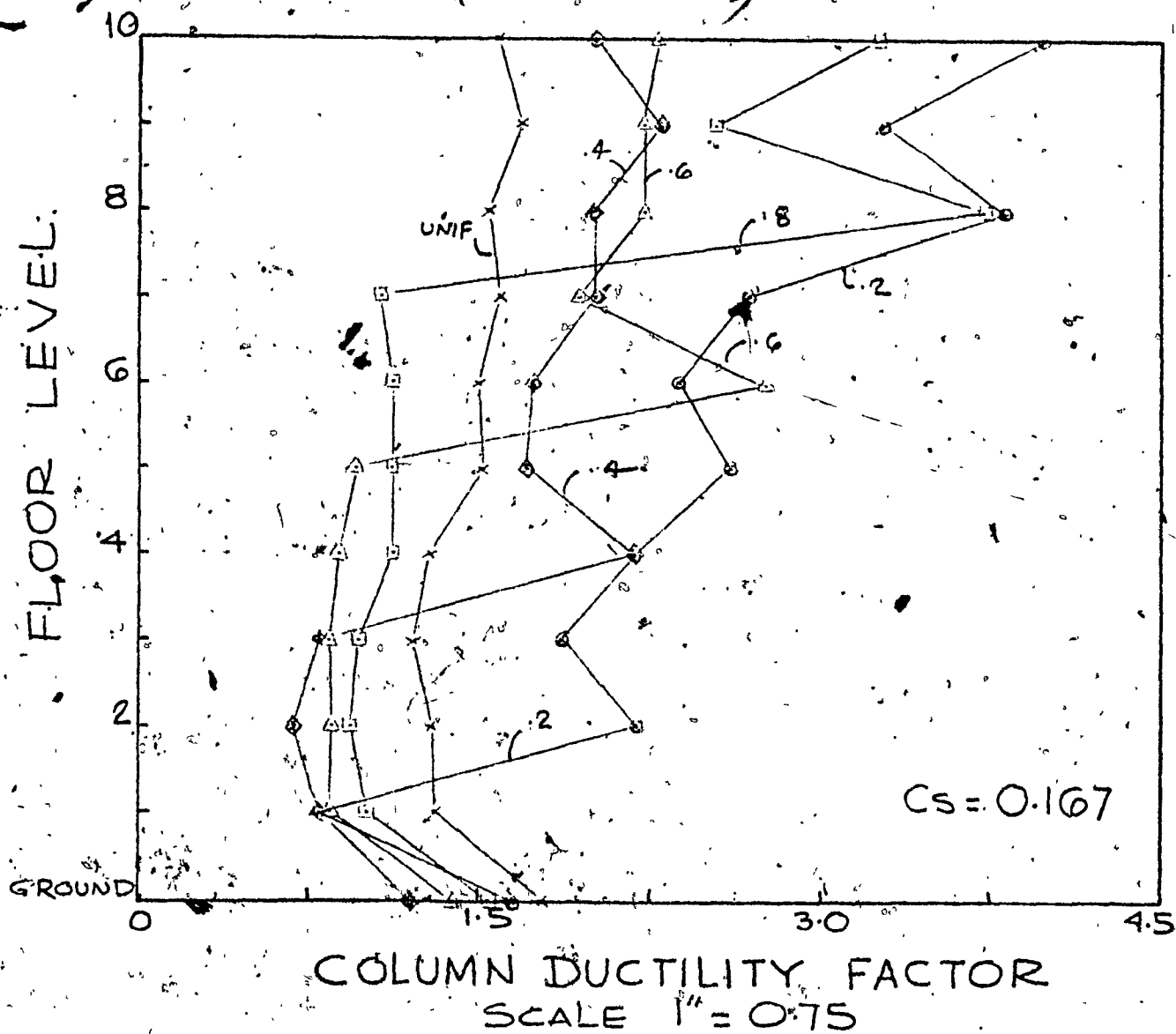
UNIFORM x — x  
 ls=0.8 □ — □  
 ls=0.6 △ — △  
 ls=0.4 ◇ — ◇  
 ls=0.2 • — •



- UNIFORM x
- I<sub>s</sub>=0.8 □
- I<sub>s</sub>=0.6 △
- I<sub>s</sub>=0.4 ◇
- I<sub>s</sub>=0.2 ○

INELASTIC RESPONSE

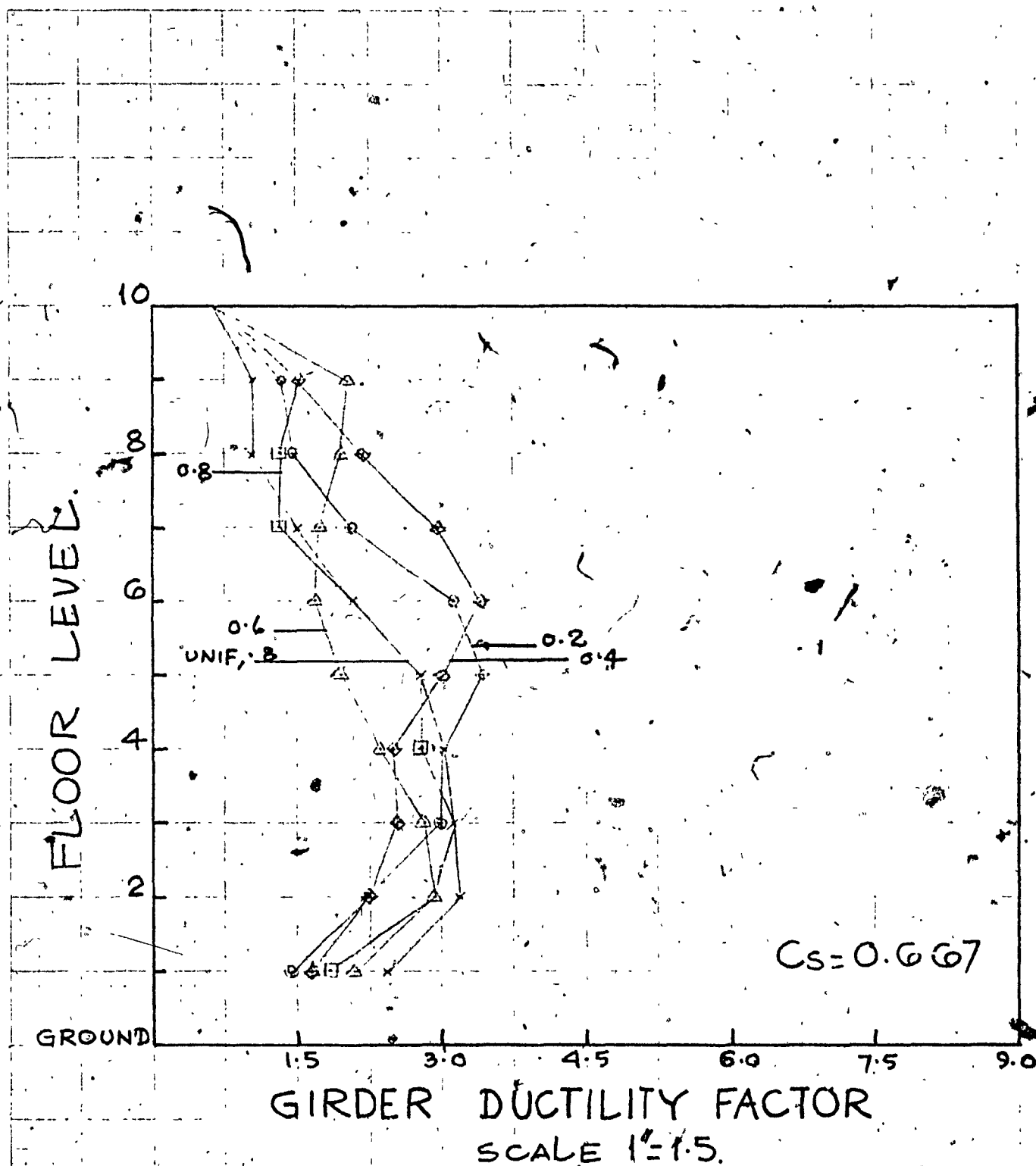
FIG. 4.8.C



ELASTIC RESPONSE

UNIFORM  $\times$ — $\times$   
 $1s=0.8$   $\square$ — $\square$   
 $1s=0.6$   $\triangle$ — $\triangle$   
 $1s=0.4$   $\diamond$ — $\diamond$   
 $1s=0.2$   $\circ$ — $\circ$

FIG. 4.8.C

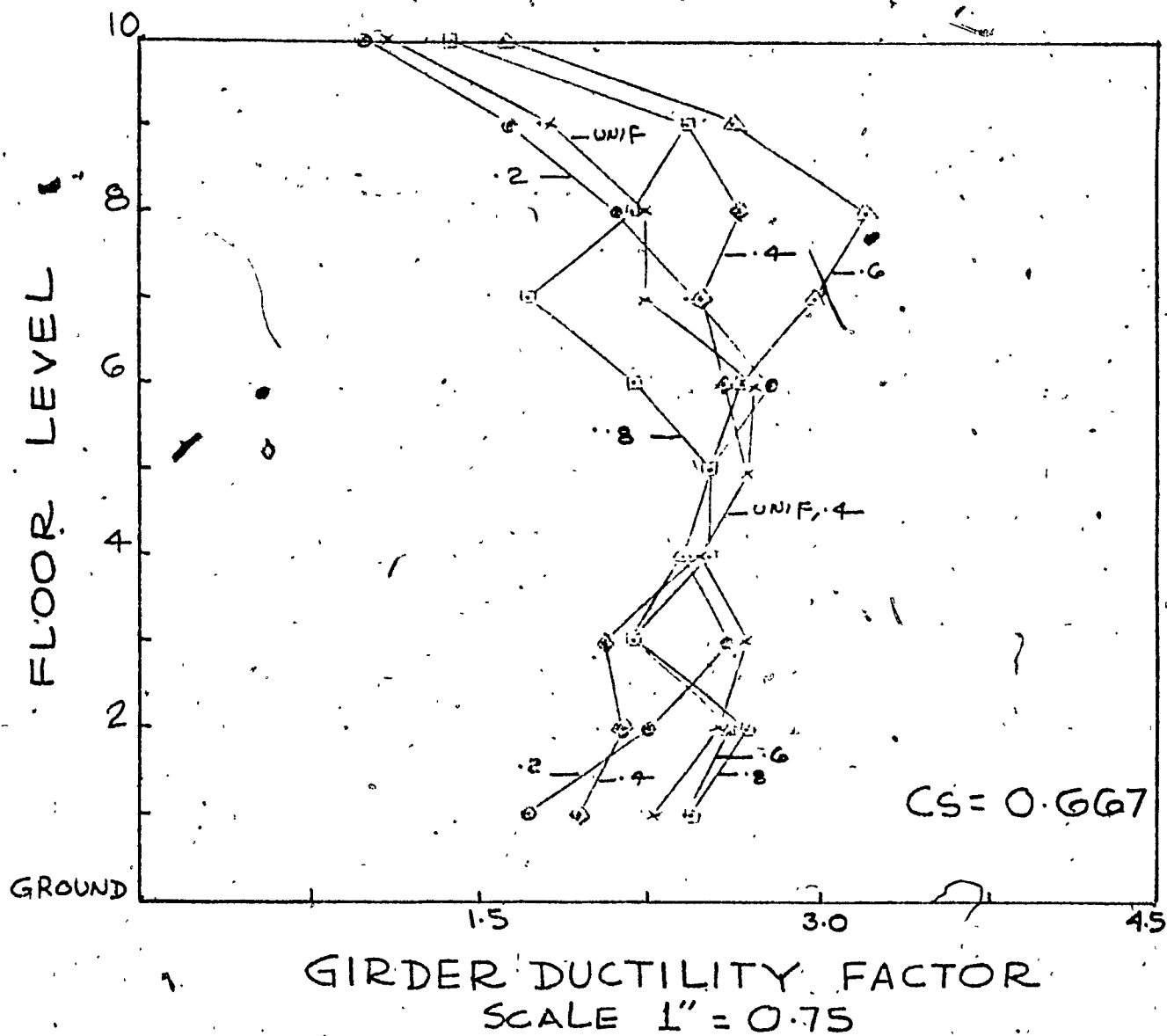


INELASTIC RESPONSE

FIG. 4.9.A

UNIFORM  
 $L_s = 0.8$   
 $L_s = 0.6$   
 $L_s = 0.4$   
 $L_s = 0.2$

—X—X—  
 —□—□—  
 —△—△—  
 —◇—◇—  
 —○—○—

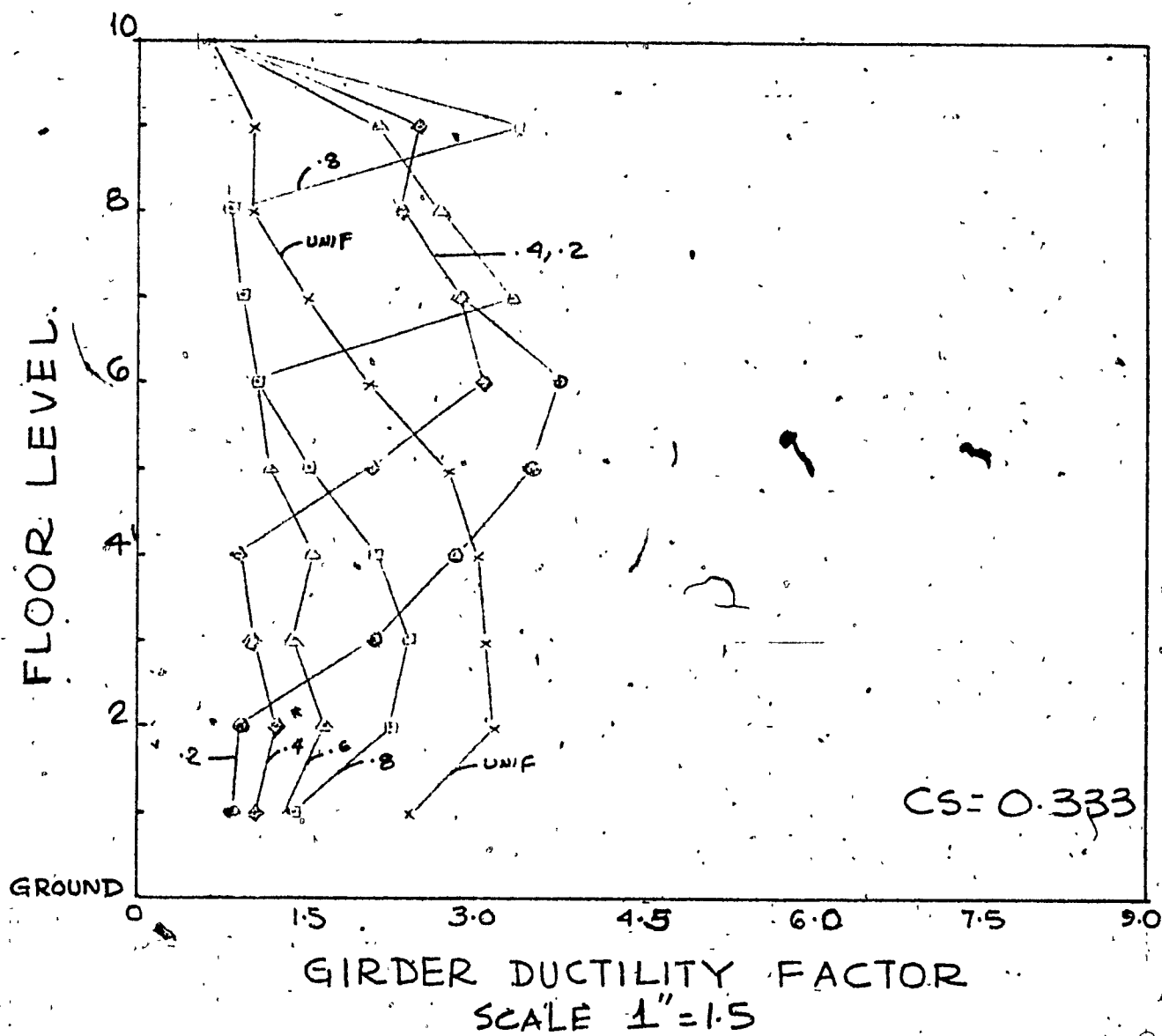


ELASTIC RESPONSE

- UNIFORM
- ls:0.8
- ls:0.6
- ls:0.4
- ls:0.2

FIG 4.9.A





INELASTIC RESPONSE

UNIFORM  $\times$ — $\times$

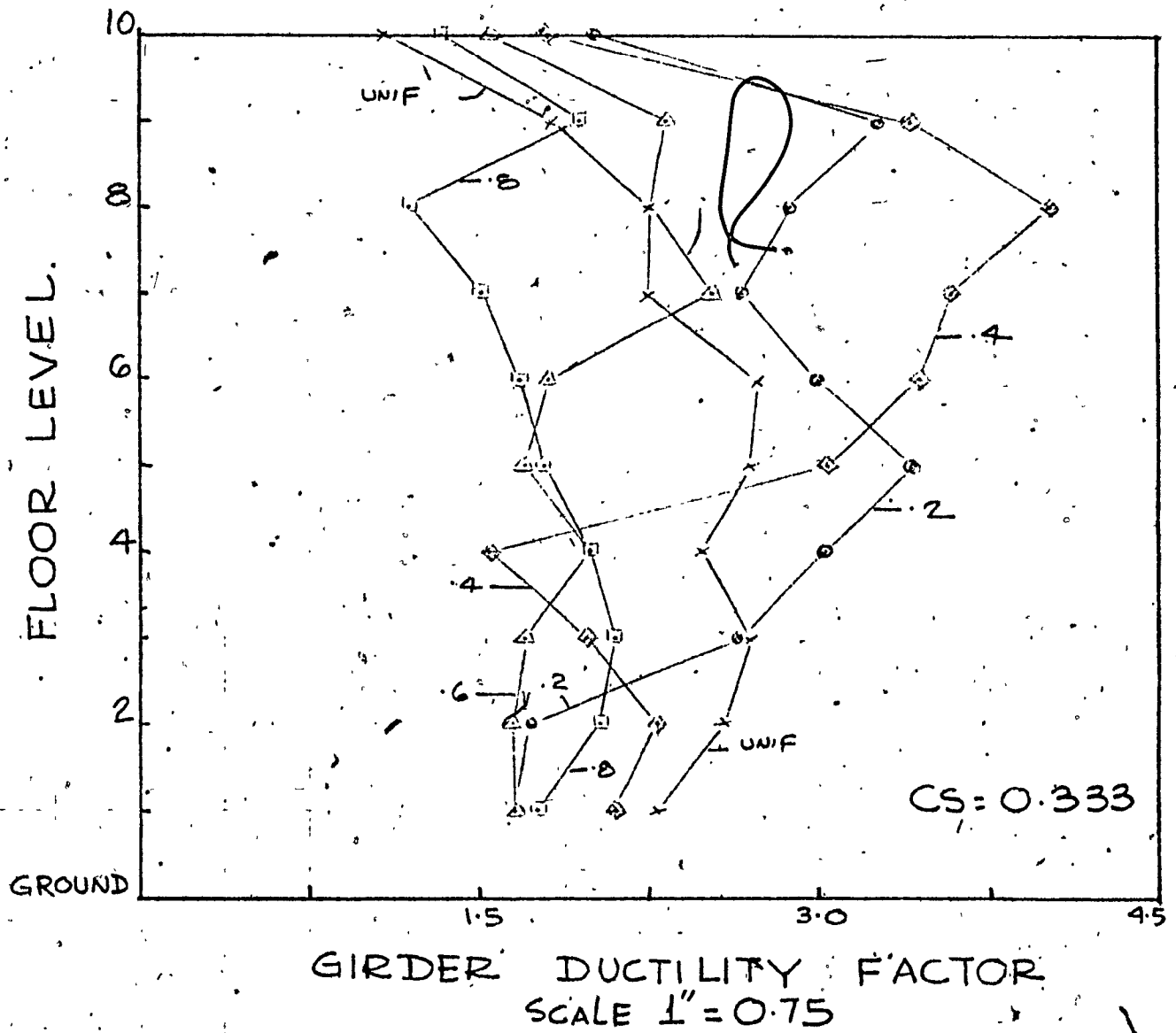
$l_s = 0.8$   $\square$ — $\square$

$l_s = 0.6$   $\triangle$ — $\triangle$

$l_s = 0.4$   $\diamond$ — $\diamond$

$l_s = 0.2$   $\oplus$ — $\oplus$

FIG. 4.9.B



ELASTIC RESPONSE

UNIFORM  $\times$ — $\times$

$ls=0.8$   $\square$ — $\square$

$ls=0.6$   $\triangle$ — $\triangle$

$ls=0.4$   $\circ$ — $\circ$

$ls=0.2$   $\bullet$ — $\bullet$

FIG. 4.9.B

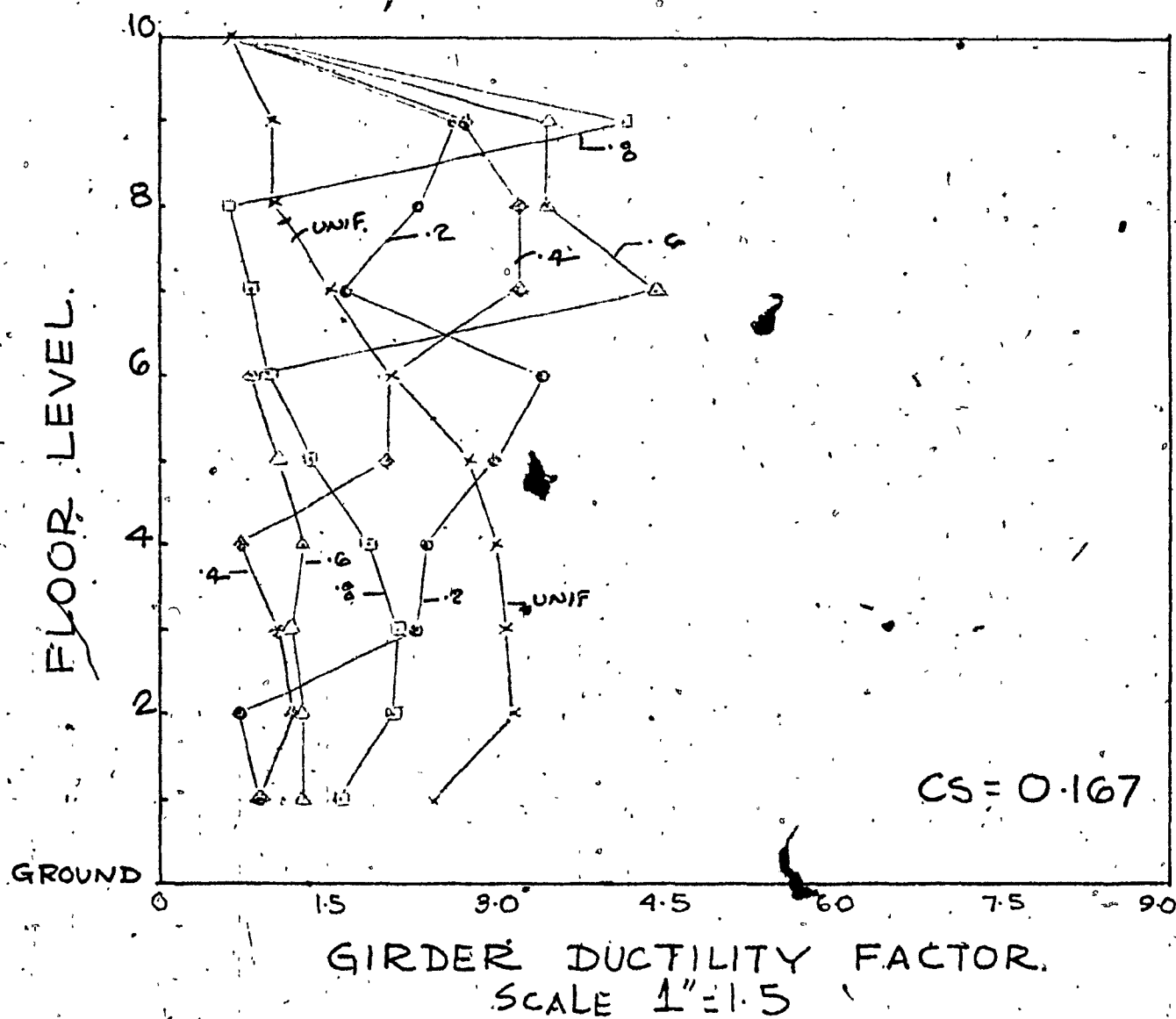
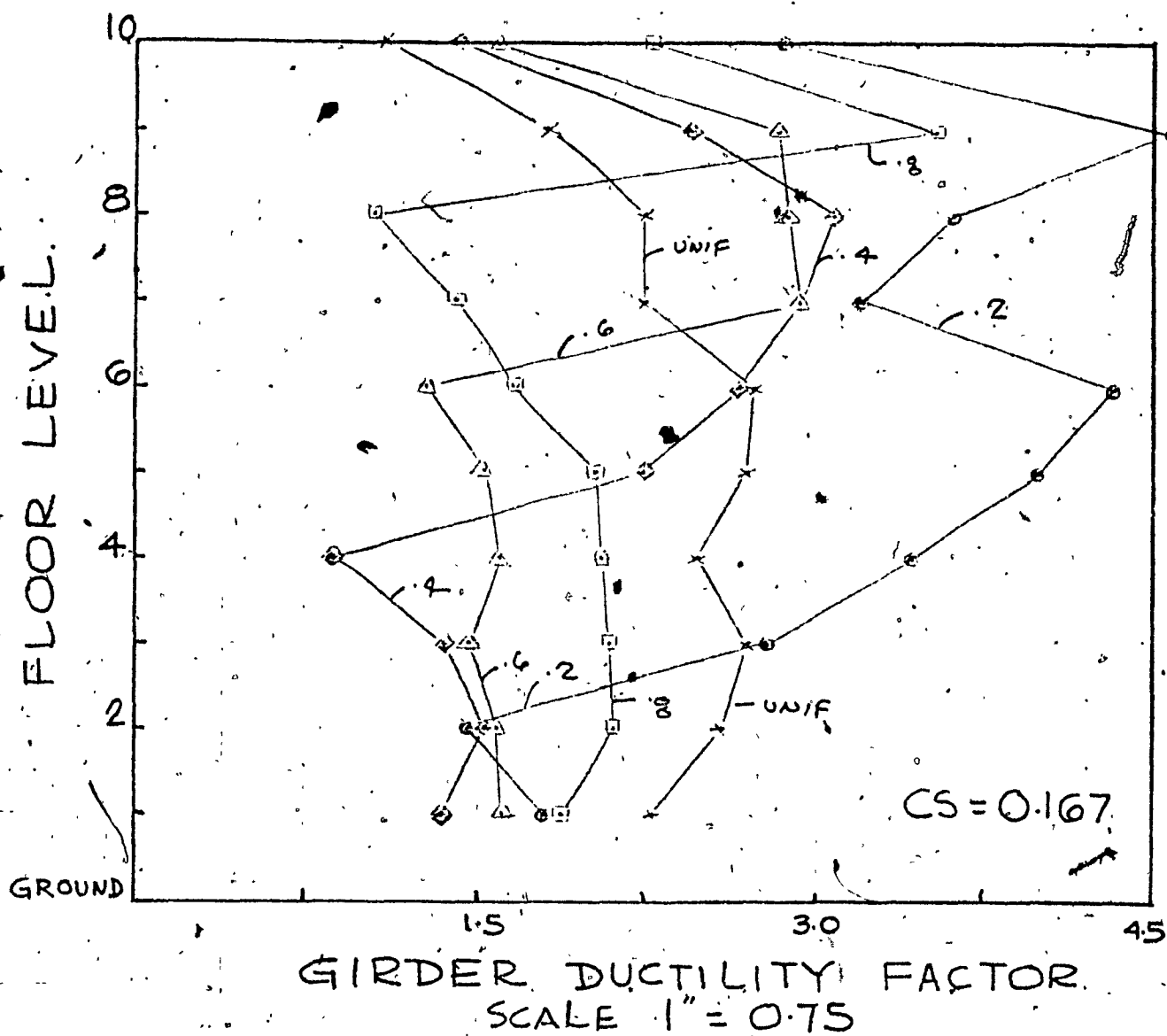


FIG. 4.9.C



ELASTIC RESPONSE

UNIFORM  $\times$  —  $\times$   
 $l_s = 0.8$   $\square$  —  $\square$   
 $l_s = 0.6$   $\triangle$  —  $\triangle$   
 $l_s = 0.4$   $\diamond$  —  $\diamond$   
 $l_s = 0.2$   $\circ$  —  $\circ$

FIG. 4.9.C

# REFERENCES

1. SEAOC, Recommended Lateral Force Requirements and Commentary, 1967 edition.
2. U.B.C., International Conference of Bldg. Officials, 1970 edition.
3. American National Standard, ANSI, A58.1-1972.
4. National Bldg. Code, Recommended by American Insurance Association, 1967 edition.
5. N.B.C., Canada, 1970.
6. Penzien, J., "Earthquake Response of Irregularly shaped Bldgs.", Proc. Fourth World Conference Earthquake Engr., Santiago, Chile, 1969.
7. Berg, G.V., "Earthquake stresses in Tall Bldgs. with Setback", Proc. of 2nd Sym. on E.Q. Engr., University of Roorkee, India, 1962.
8. Jhaveri, D.P., "Earthquake Forces in Tall Bldgs. with Setbacks", Ph.D Thesis, University of Mich., 1967.
9. Pekau, O.A., "Inelastic Behaviour of Frame Structures under static and E.Q. Forces", Ph.D Thesis, University of Waterloo, 1970.
10. Pekau, O.A., and Green, "Inelastic Structures with Setbacks", Proc. Fifth World Conference Earthquake Engr., Rome, Italy, 1973.
11. Pekau, O.A., "Computer Programme for the Inelastic Response of Multistorey Frame Structures, Subjected to E.Q. Excitation", Report

prepared for Canada Emergency Measures Organization, 1970.

12. Bustamante, J.I., and Rapoport, "Momento de Volteo y Fuerzas Cortantes Sismicas", Bol. Soc. Mex. Ing. Sism., 1964.